UNH Analytical Modeling of the Interstellar Helium Phase-Space and IBEX Intensity Distribution

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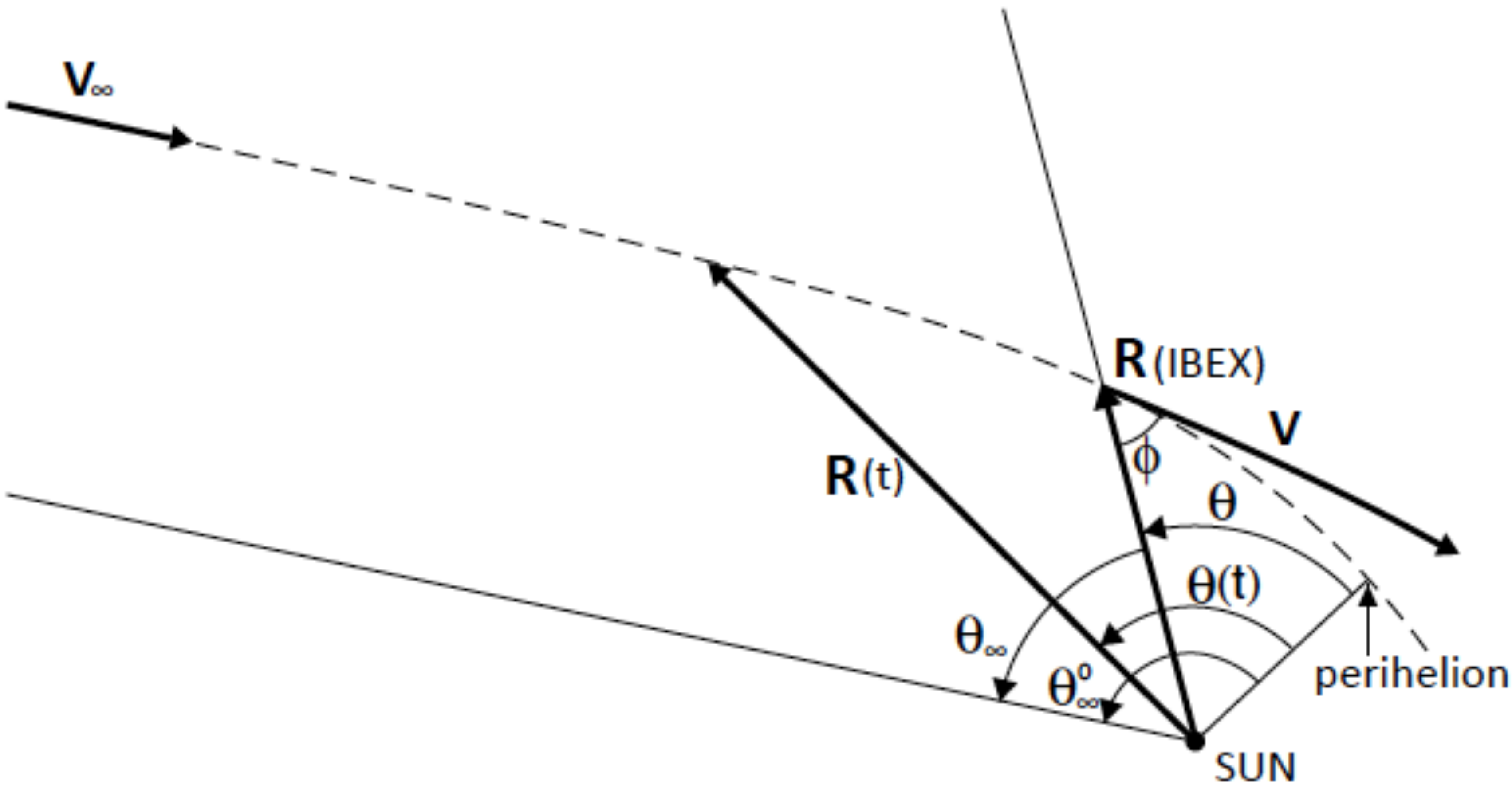
Geometry



V_∞
R (IBEX)
V
R (t)
ϕ
θ
θ(t)
θ_∞
θ_∞^0
perihelion
SUN

Basic Equations

$$\frac{1}{R} = \frac{mk}{l^2}\left(1 + \varepsilon \cos\theta\right)$$

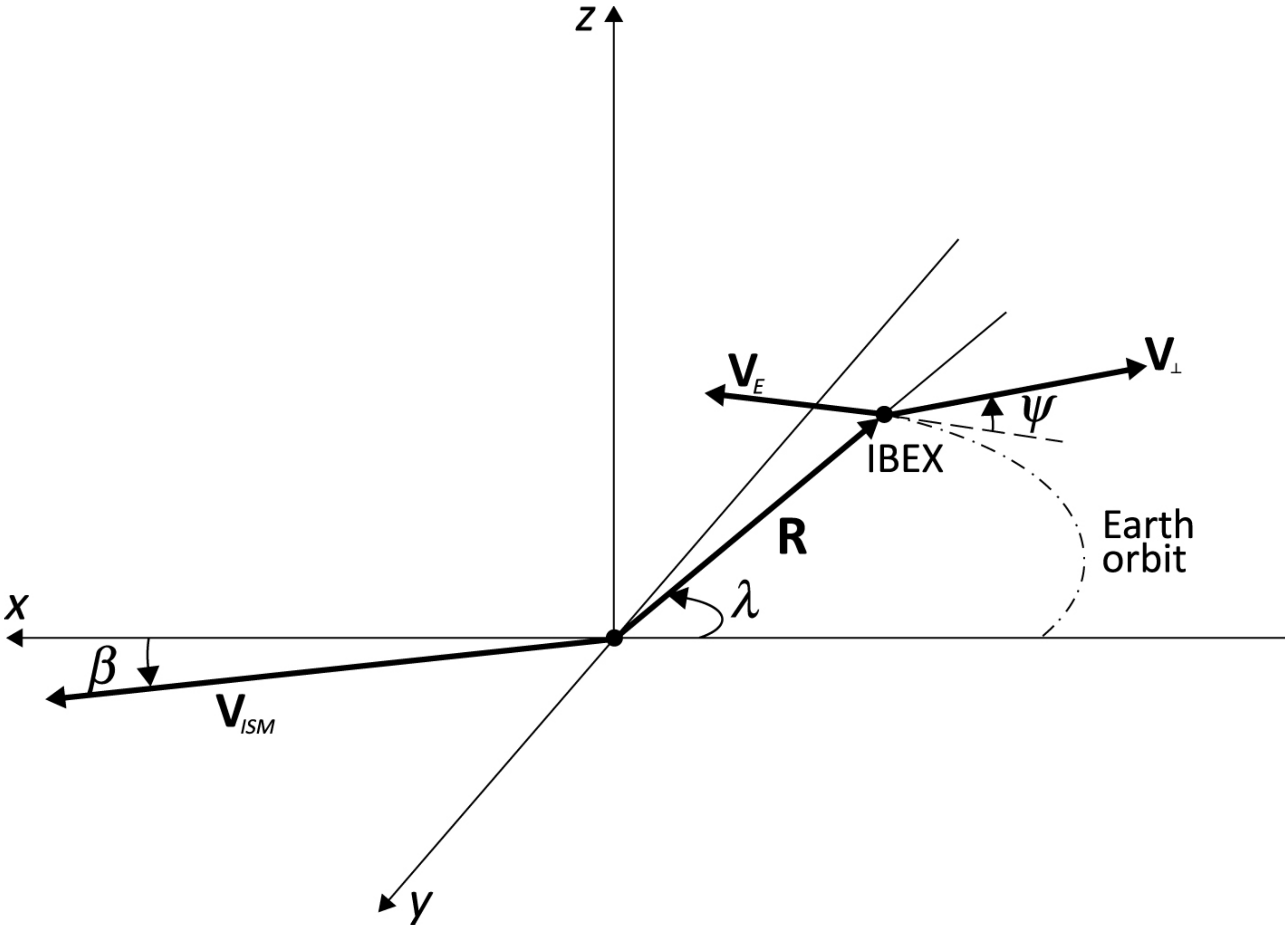
$$l = mRV\sin\phi$$

$$\varepsilon^2 = 1 + \frac{2l^2 E}{mk^2}$$

$$E = \frac{1}{2}mV^2 - \frac{k}{R}$$

$$k = GmM_s = mR_E V_E^2$$

(For Circular Earth Orbit)



z
x
y
$\mathbf{V}_E$
$\mathbf{V}_\perp$
ψ
IBEX
R
Earth
orbit
λ
β
$\mathbf{V}_{ISM}$

ISM Helium Distribution

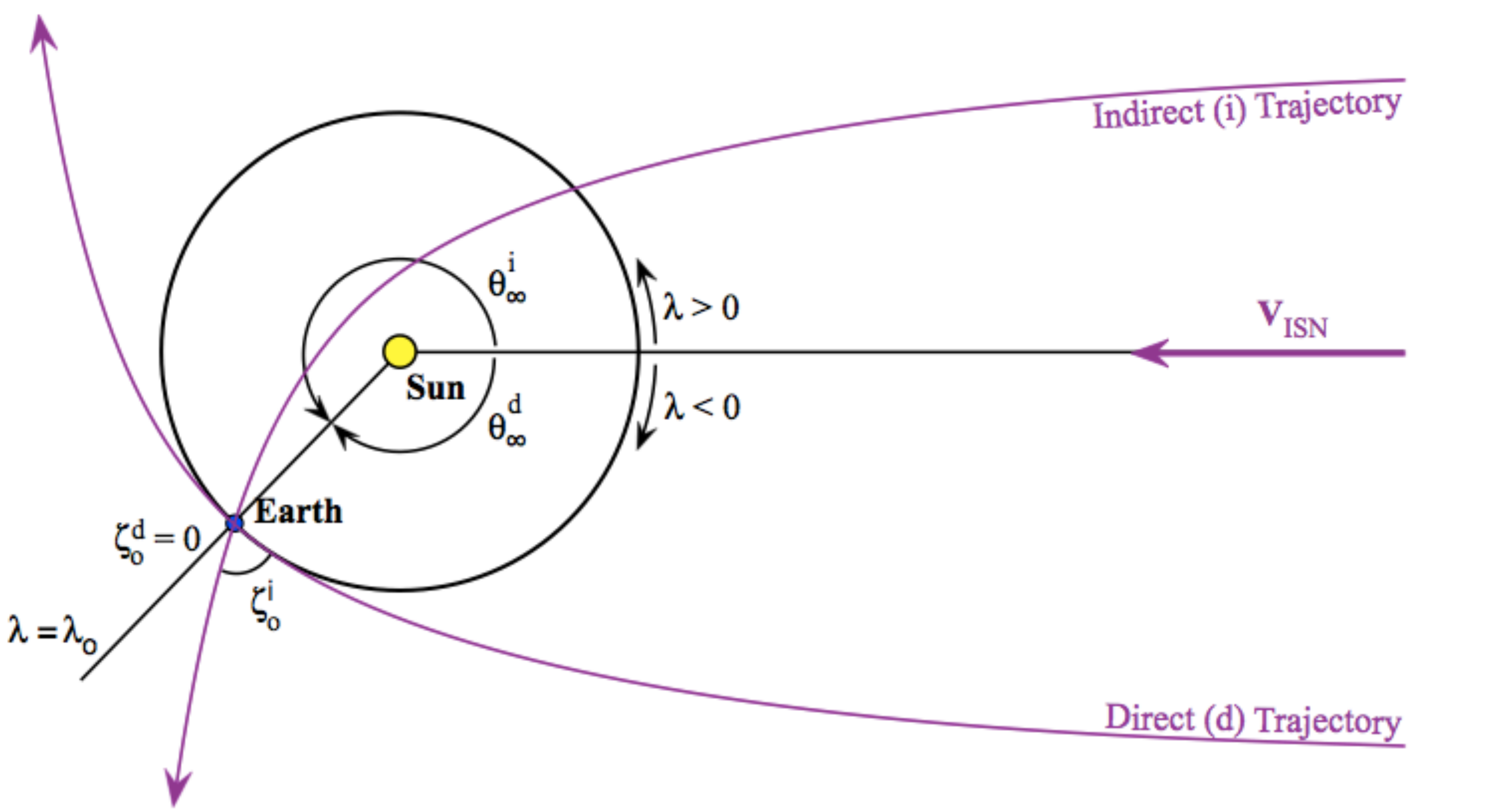
$$\mathbf{V}_\infty = V_\infty \cos\theta_\infty \left(\mathbf{i}\cos\lambda + \mathbf{j}\sin\lambda\right) + V_\infty \sin\theta_\infty \left[\cos\psi\left(-\mathbf{i}\sin\lambda + \mathbf{j}\cos\lambda\right) + \mathbf{k}\sin\psi\right]$$

$$f(\mathbf{V}_\infty) = n\left(2\pi k_B T / m\right)^{-3/2} \exp\left[-\,|\,\mathbf{V}_\infty - \mathbf{V}_{ISM}\,|^2 \left(2k_B T / m\right)^{-1}\right]$$

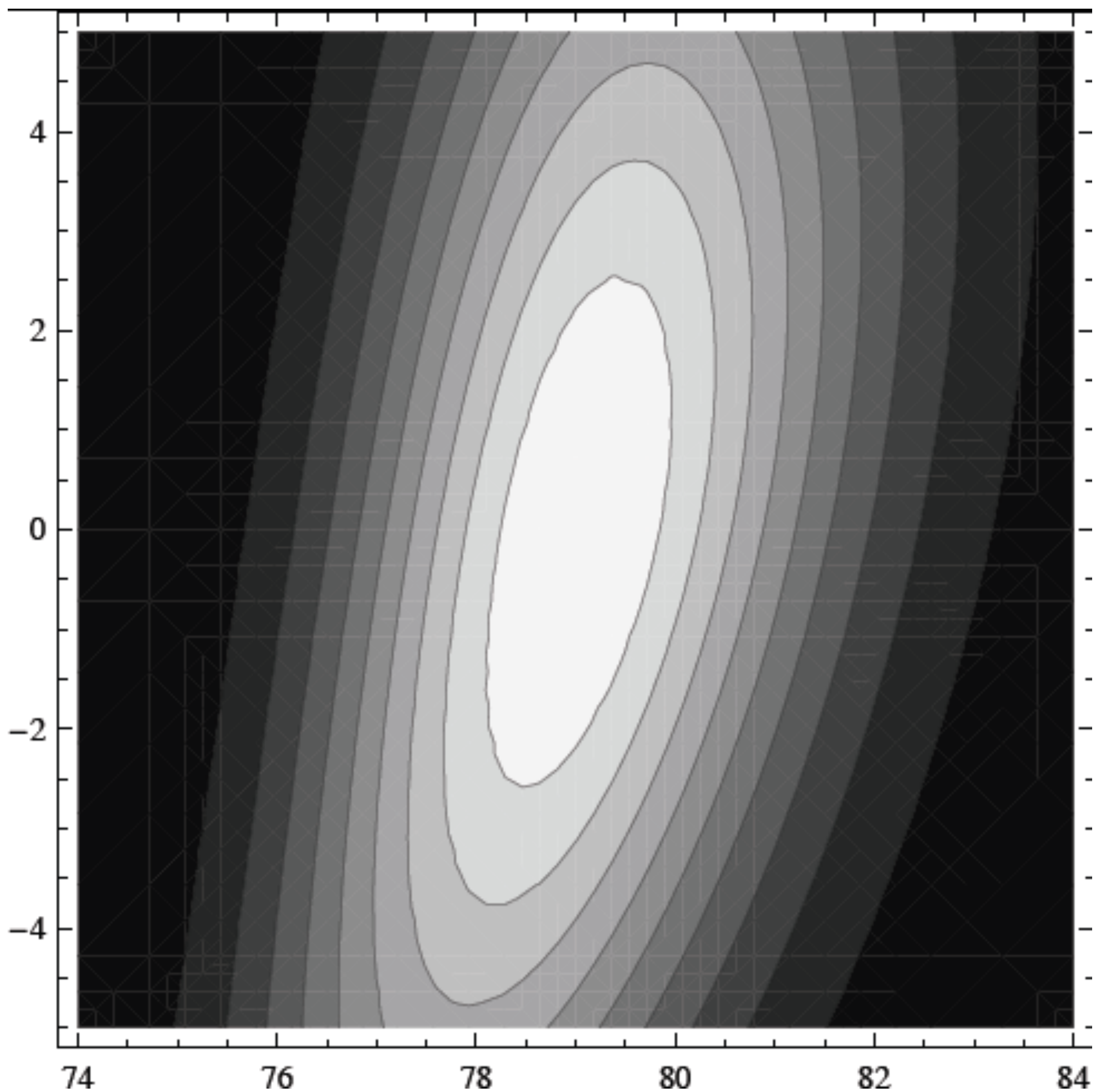
$$f(v,\psi,\phi-\pi/2)(V_E^3/n)(\pi t)^{3/2} = \exp\Big\{-t^{-1}\Big[v^2 - 2 + v_{ISM}^2 - 2v_{ISM}(v^2-2)^{1/2} \times$$

$$\times\left(\cos\theta_\infty\cos\beta\cos\lambda - \sin\theta_\infty[\cos\beta\cos\psi\sin\lambda + \sin\beta\sin\psi]\right)\Big]\Big\}$$

Direct & Indirect Trajectories and the "Sweet Spot"



Exact f in Ecliptic at “Sweet Spot” for $\boldsymbol{\beta = 0}$



Expansion About Peak

$$\mathbf{v}_0(\lambda) = v_{\perp 0}(\lambda)\mathbf{e}_\perp + v_{r0}(\lambda)\mathbf{e}_r + v_{z0}(\lambda)\mathbf{e}_z$$

$$v_\perp = v_{\perp 0} + \delta v_\perp; \quad v_r = v_{r0} + \delta v_r; \quad v_z = v_{z0} + \delta v_z$$

$$f(v,\psi,\phi - \pi/2)(V_E^3/n)(\pi t)^{3/2} = \exp\left\{ -\frac{1}{2} D_{v_\perp v_\perp} (\delta v_\perp)^2 \right.$$

$$\left. -\frac{1}{2} D_{v_r v_r} (\delta v_r)^2 - \frac{1}{2} D_{v_z v_z} (\delta v_z)^2 + D_{v_\perp v_r} (\delta v_\perp \delta v_r) + D_{v_\perp v_z} (\delta v_\perp \delta v_z) + D_{v_r v_z} (\delta v_r \delta v_z) \right\}$$

Velocity at the Peak: Solar and IBEX Frames

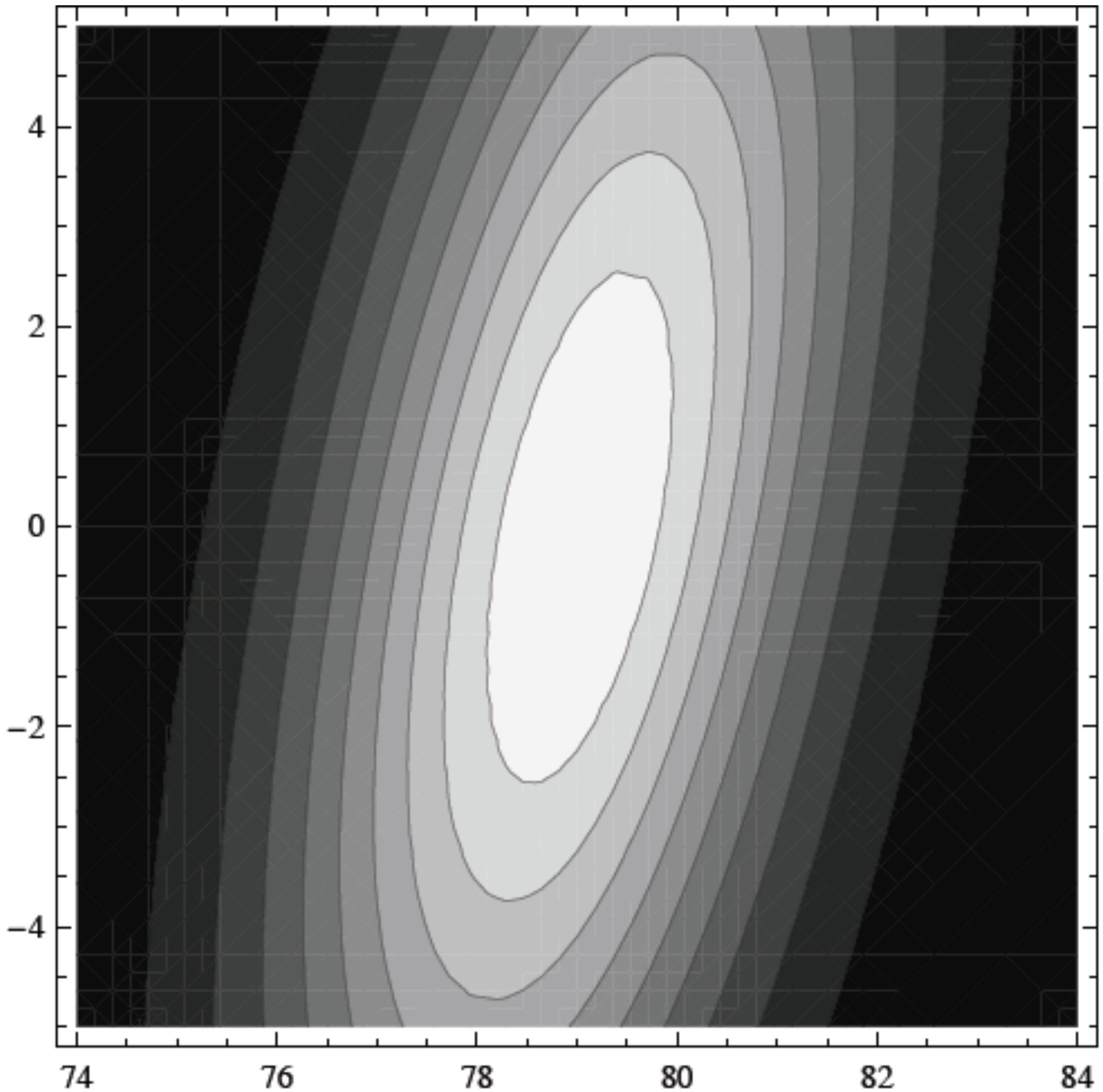
1. $v_0^2 = 2 + v_{ISM}^2$
2. $\tan\psi_0 = \tan\beta / \sin\lambda$
3. $v_0 \cos\zeta_0 = \pm(v_{ISM}/2)\left(1-\cos^2\beta\cos^2\lambda\right)^{1/2} +$
   $+\left[v_{ISM}^2\left(1-\cos^2\beta\cos^2\lambda\right)+4\left(1-\cos\beta\cos\lambda\right)\right]^{1/2}/2$

$$v_{\perp 0} = v_0 \cos\zeta_0 \cos\psi_0 + 1$$

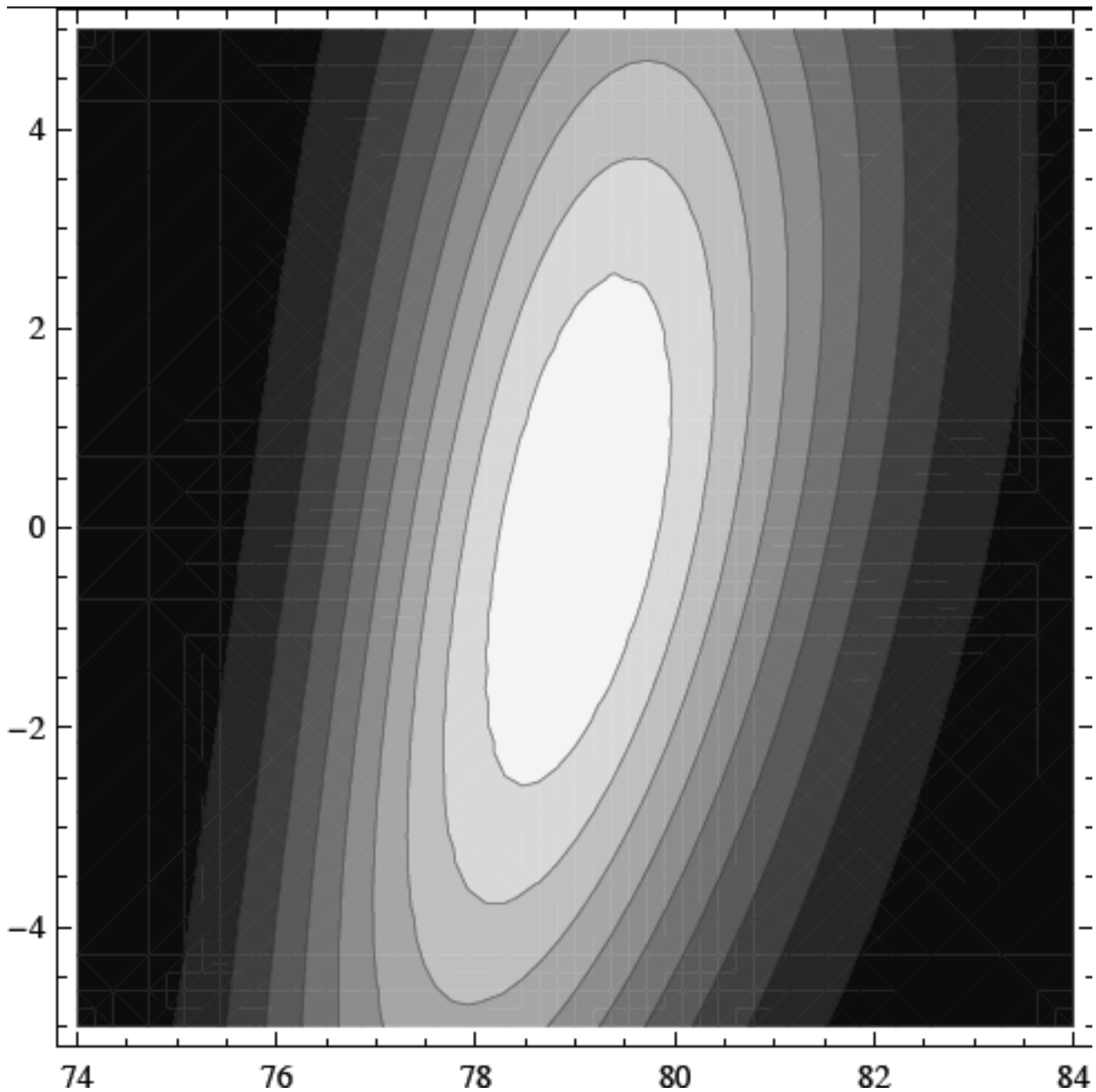
$$v_{r0} = v_0 \sin\zeta_0$$

$$v_{z0} = v_0 \cos\zeta_0 \sin\psi_0$$

F to 2nd Order in Ecliptic at "Sweet Spot" for β=0



Compare with Exact Distribution



Expansion Distribution at the "Sweet Spot" for β = 0

$$f(v,\psi,\phi-\pi/2)(V_E^3/n)(\pi t)^{3/2} \simeq$$

$$\exp\left\{-\frac{1}{2}D_{v_z v_z}(v_z)^2-\frac{1}{2}D_{v_\perp v_\perp}(v_\perp-v_{\perp 0})^2-\frac{1}{2}D_{v_r v_r}(v_r)^2+D_{v_\perp v_r}(v_\perp-v_{\perp 0})(v_r)\right\}$$

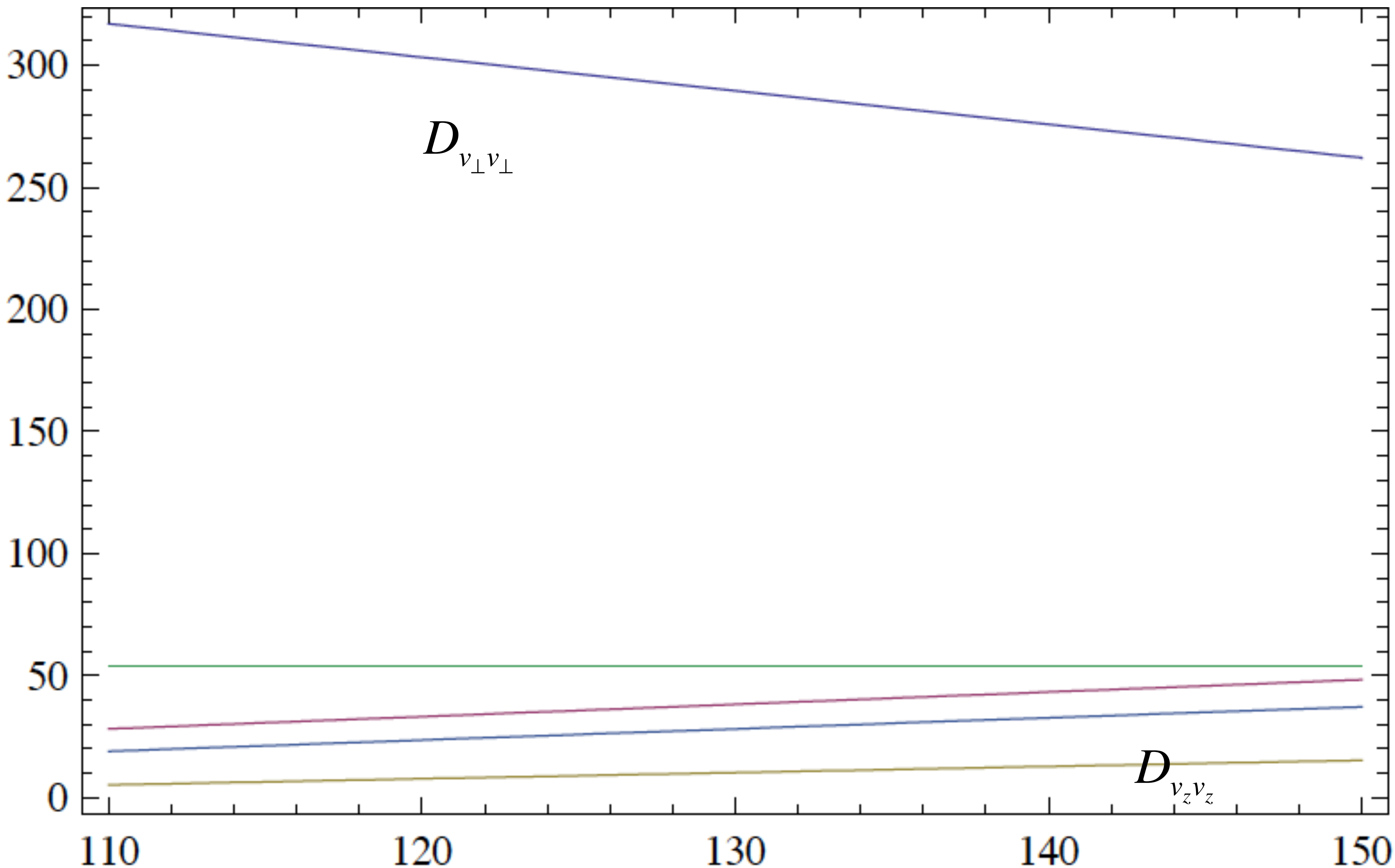
$$D_{v_z v_z}=t^{-1}2v_{ISM}^4\left(1+v_{ISM}^2\right)^{-2} \qquad \leftarrow \text{Smallest}$$

$$D_{v_\perp v_\perp}=t^{-1}\left[2+4v_{ISM}^{-2}+8\left(1+v_{ISM}^2\right)^{-2}\right]$$

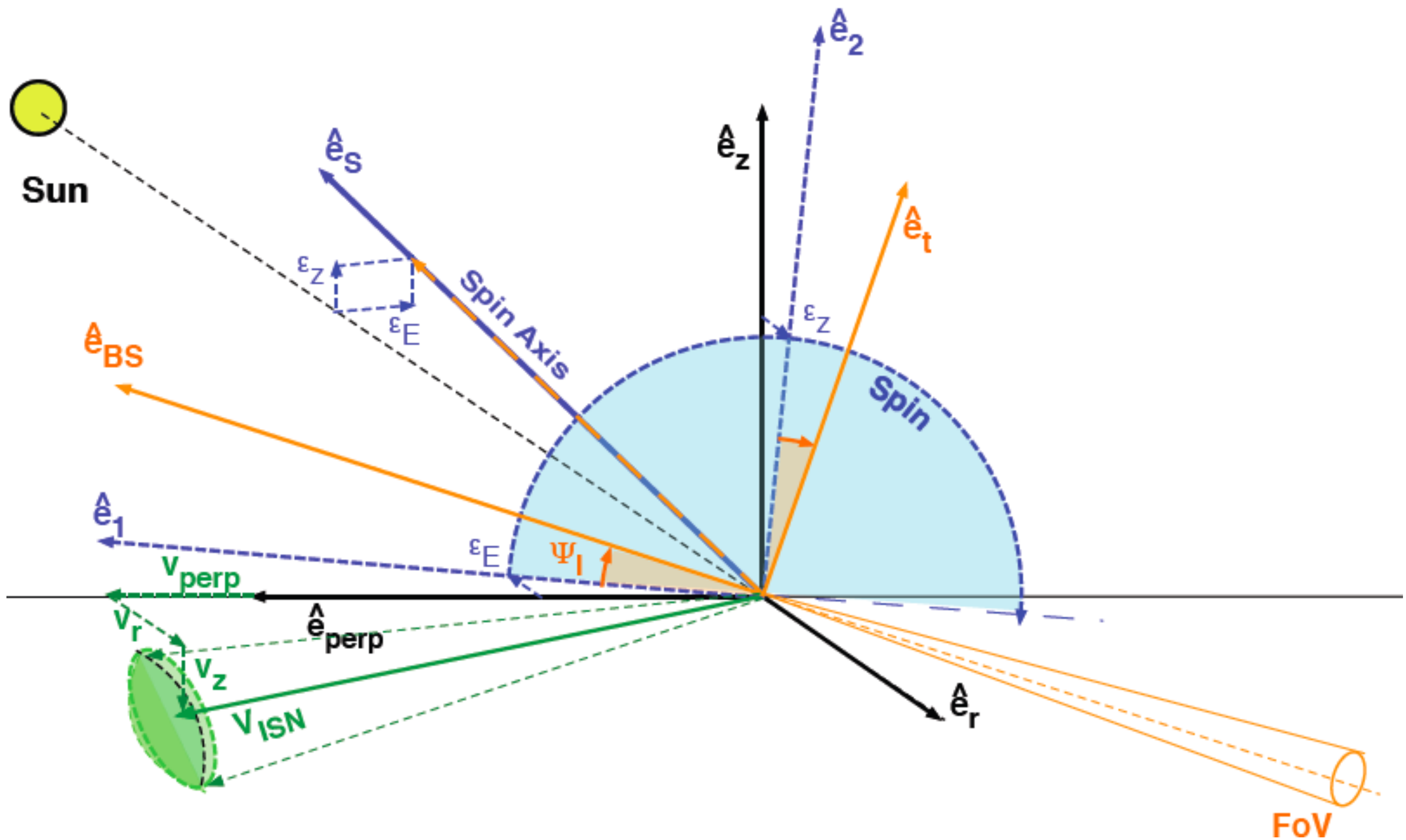
$$D_{v_r v_r}=t^{-1}2v_{ISM}^2\left(2+v_{ISM}^2\right)\left(1+v_{ISM}^2\right)^{-2}$$

$$D_{v_\perp v_r}=t^{-1}4v_{ISM}\left(2+v_{ISM}^2\right)^{1/2}\left(1+v_{ISM}^2\right)^{-2}$$

D Values vs Longitude



Spin-axis Geometry



IBEX Count Rate

$$C(\psi_I,\lambda)\propto\int_{-\infty}^{\infty} dv_{BS}\, dv_s\, dv_t v_{BS}\exp\left(-\frac{1}{2\sigma_I^2}\frac{v_s^2+v_t^2}{v_{BS}^2}\right)\exp\left(-P\right)\times$$

$$\times\exp\left\{-\frac{1}{2}D_{v_{BS}v_{BS}}(\delta v_{BS})^2-\frac{1}{2}D_{v_s v_s}(\delta v_s)^2-\frac{1}{2}D_{v_t v_t}(\delta v_t)^2+\right.$$

$$+D_{v_{BS}v_s}\delta v_{BS}\delta v_s+D_{v_{BS}v_t}\delta v_{BS}\delta v_t+D_{v_t v_s}\delta v_t\delta v_s+$$

$$\left.+C_{v_{BS}}\delta v_{BS}+C_{v_s}\delta v_s+C_{v_t}\delta v_t+C_0\right\}$$

Result to Third Order:

$$v_{z0} \sim v_{r0} \sim D_{v_r v_z} \sim D_{v_\perp v_z} \sim \psi_I \ll 1$$

$$C = C_0 \sigma_I^2 \exp\left\{-\frac{1}{2}\left(1-\sigma_I^2 v_{\perp 0}^2\right)\left[D_{v_z v_z}^2\left(v_{z0}-\psi_I v_{\perp 0}\right)^2 + \overline{D}^2\left(v_{r0}+\varepsilon_E v_{\perp 0}\right)^2\right]\right\}\exp\left(-P_0\right)\times$$

$$\times\exp\left\{-\frac{1}{2}\left[-\frac{2D_{v_\perp v_r}}{D_{v_\perp v_\perp}}\overline{D}\varepsilon_E\left(v_{r0}+v_{\perp 0}\varepsilon_E\right)^2 - \overline{D}2v_{\perp 0}\varepsilon_z\psi_I\left(v_{r0}+v_{\perp 0}\varepsilon_E\right) - \frac{2D_{v_\perp v_r}}{D_{v_\perp v_\perp}}\varepsilon_E D_{v_z v_z}\left(v_{z0}-v_{\perp 0}\psi_I\right)^2 +\right.\right.$$

$$\left.\left. + \left(\frac{D_{v_\perp v_r}}{D_{v_\perp v_\perp}}D_{v_\perp v_z} + D_{v_r v_z}\right)2\left(v_{r0}+v_{\perp 0}\varepsilon_E\right)\left(v_{\perp 0}\psi_I - v_{z0}\right) - D_{v_z v_z}\frac{D_{v_\perp v_r}}{D_{v_\perp v_\perp}}2\left(v_{\perp 0}\psi_I - v_{z0}\right)\left(v_{z0}\varepsilon_E + v_{r0}\psi_I\right)\right]\right\}$$

$$\overline{D} = D_{v_r v_r} - \left(D_{v_r v_\perp}\right)^2 / D_{v_\perp v_\perp}$$

Integrate over Spin Angle:

$$v_{z0} \sim v_{r0} \sim D_{v_r v_z} \sim D_{v_\perp v_z} \sim \psi_I \ll 1$$

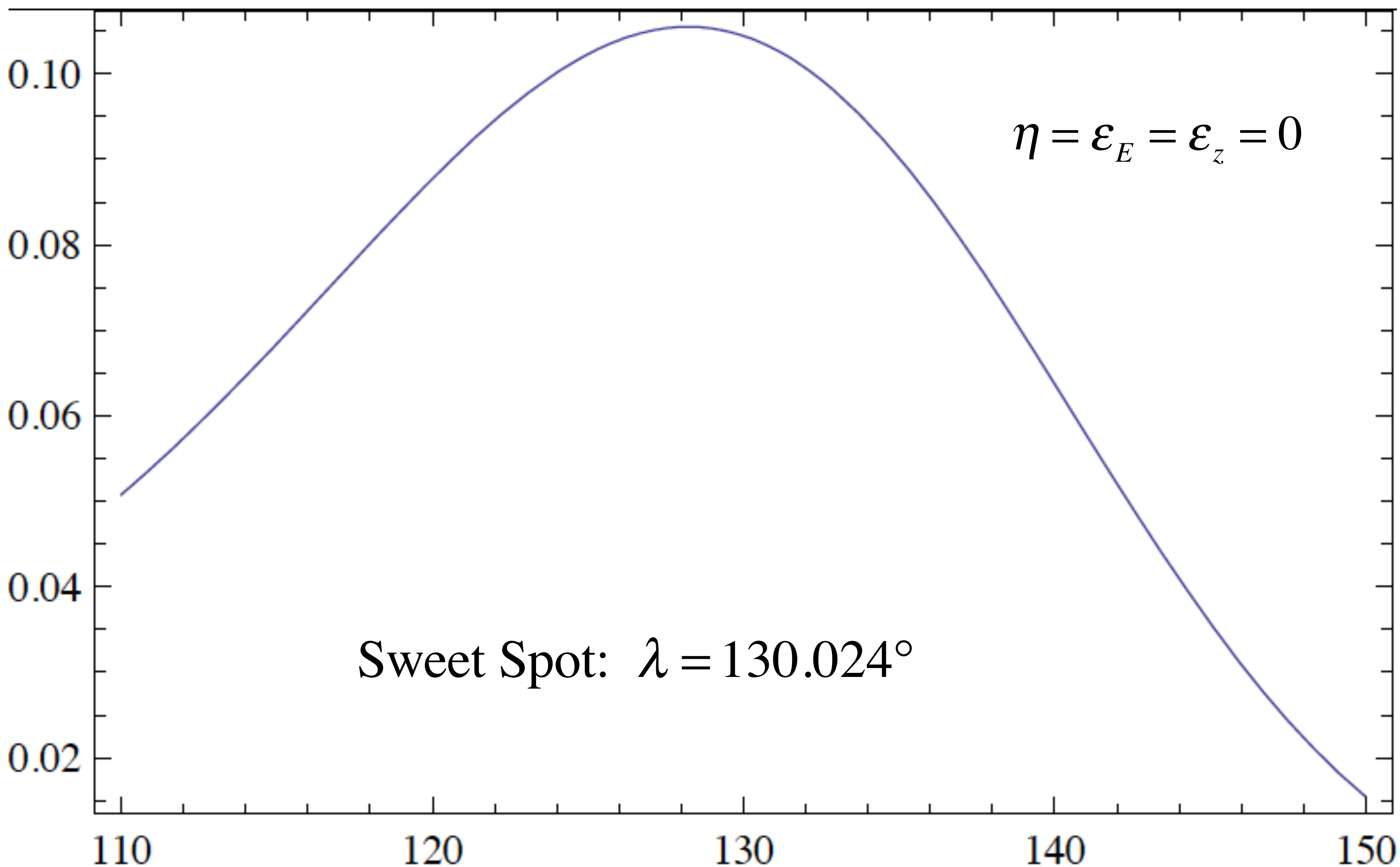
$$C \propto v_{\perp 0}^2 \sigma_I^2 \left[D_{v_\perp v_\perp} D_{v_z v_z} \left(1 + D_{v_r v_r} \sigma_I^2 v_{\perp 0}^2\right)\left(1 - D_{v_z v_z}^2 \sigma_I^4 v_{\perp 0}^4\right)\right]^{1/2} \times$$

$$\times \exp\left\{ \frac{1}{2} \sigma_I^2 v_{\perp 0}^2 \overline{D}^2 \left(v_{r0} + \varepsilon_E v_{\perp 0}\right)^2 \right\} \times$$

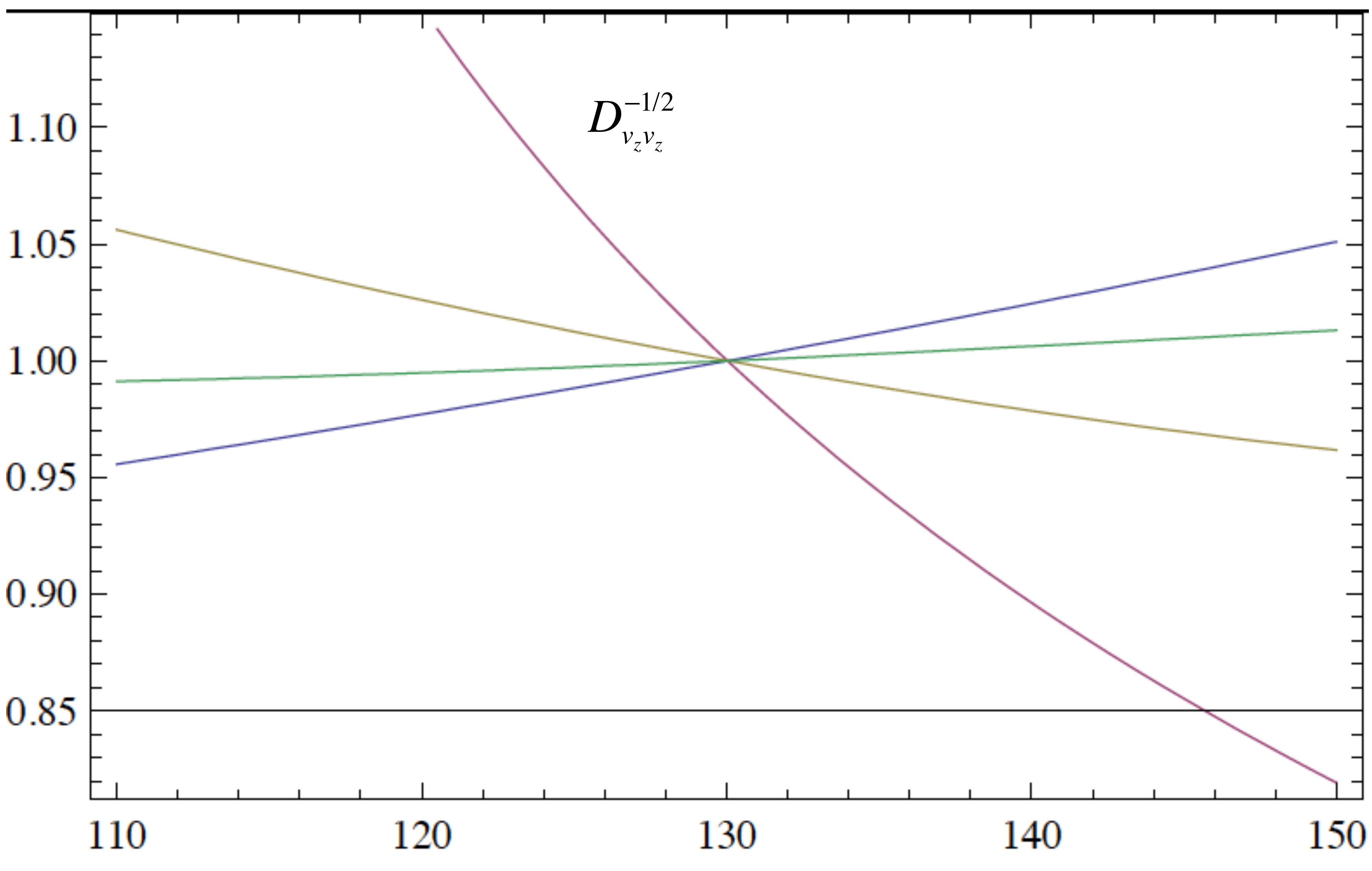
$$\times \exp\left\{ -\frac{\overline{D}}{2} \left[\left(v_{r0} + v_{\perp 0}\varepsilon_E - v_{z0}\varepsilon_z\right)^2 - \left(v_{r0} + v_{\perp 0}\varepsilon_E\right)^2 \frac{2D_{v_\perp v_r}}{D_{v_\perp v_\perp}} \varepsilon_E \right] \right\}$$

$$\overline{D} = D_{v_r v_r} - \left(D_{v_r v_\perp}\right)^2 / D_{v_\perp v_\perp}$$

Intensity vs Longitude



Correction Factors



Behavior in Focusing Cone

$$D_{v_z v_z} = \frac{2v_{ISM}^2}{t(2+v_{ISM}^2)}\left[\frac{v_{ISM}^2(2+v_{ISM}^2)}{(1+v_{ISM}^2)^2} + 2\frac{\cos\beta(\cos\lambda - \cos\lambda_0)}{(1+v_{ISM}^2)} - \right.$$

$$\left. - \cos^2\beta(\cos\lambda - \cos\lambda_0)^2\right]$$

$$D_{v_z v_z} \to 0 \quad \text{as} \quad \lambda \to -\pi$$