Forward modeling of primary and secondary neutral helium in the heliosphere

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Modeling neutral helium as Keplerian trajectories: Basics

The movement (=trajectory) of neutral particles in the heliosphere is describable as **Kepler orbit**, as the force acting on it is a **central force** (solar gravity, minus outward radiation pressure).

Single particle: The trajectory is confined to a **plane**.

In the case of **helium** and heavier species, radiation pressure negligible and the central potential is **time independent**. (For **H** and **D**, the radiation pressure becomes important, which is time- and velocity dependent.)

=> there are **conserved quantities** constant along the entire trajectory, namely:

- Total energy=kinetic+potential
- Angular momentum
- Direction of perihelion
- Eccentricity e

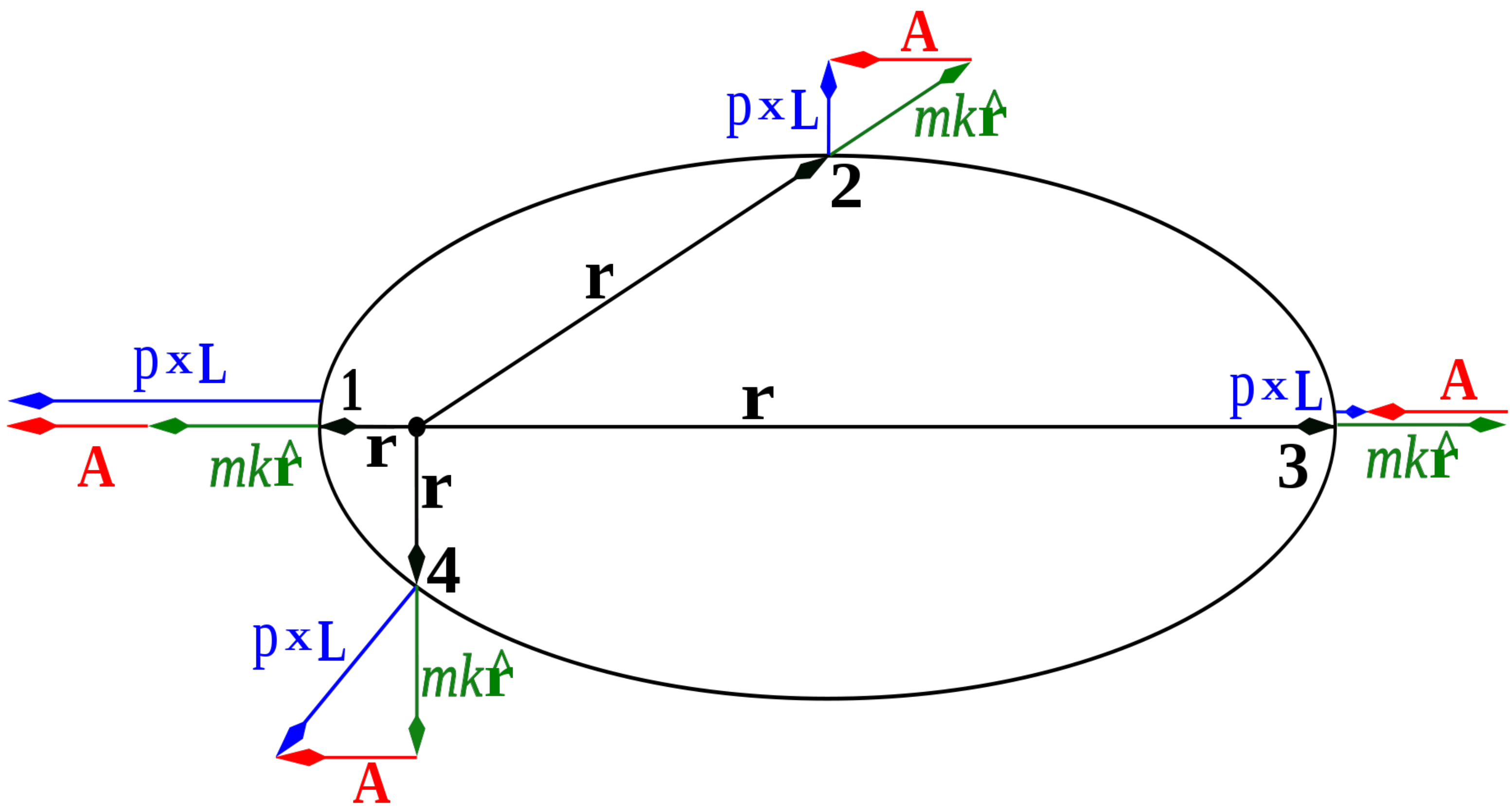
The latter two are sometimes combined into an eccentricity vector **A** (cf. Laplace-Runge-Lenz vector).

$$\vec{L} = m\,\vec{r} \times \vec{v}$$

$$\vec{A} = m\,\vec{v} \times \vec{L} - G\,M\,m\,(1-\mu)\,\hat{r}$$

$$Etot = \frac{m\,v^2}{2} - \frac{G\,M\,m\,(1-\mu)}{r}$$

Conserved eccentricity vector A (Laplace-Runge-Lenz)



The LRL vector **A** (shown in red) at four points on the elliptical orbit of a bound point particle moving under an inverse-square central force. The center of attraction is shown as a small black dot. The angular momentum vector **L** is perpendicular to the orbit. The coplanar vectors **p** × **L** and *(GMm/r)***r** are shown in blue and green, respectively. The vector **A** is constant in direction and magnitude.

Source: http://en.wikipedia.org/wiki/Laplace-Runge-Lenz_vector

Single trajectory

Same kinematics for different particle masses: use reduced conserved quantities.

$$\vec{l} = \vec{r} \times \vec{v} \quad \text{(specific angular momentum)}$$

$$\vec{a} = \vec{v} \times \vec{l} - f_\mu \hat{r} \quad \text{(eccentricity vector)}$$

$$E_{tot} = \frac{v^2}{2} - \frac{f_\mu}{r} \quad \text{(total specific energy)}$$

$f_\mu = GM_\odot(1-\mu)$ $\qquad$ $(GM_\odot = 887.6 \text{ AU km}^2 \text{ s}^{-2})$

$e = |\vec{a}|/f_\mu$

Given a set of conserved quantities, a unique trajectory is defined.

Kepler frame of reference: Inertial frame of reference with positive x-axis pointing to perihelion; x-y the orbital plane, true anomaly θ growing with time (zero at perihelion), i.e. trajectory is run through counterclockwise.

-> Only three conserved quantities are left : l_z, a_x, E_{tot}.

Kepler frame is ill defined for l_z=0 orbits (no theta variation, no plane); these straight orbits are called suncrash.

Single trajectory: peak

Type I problem ("forward calculation")
Get a full set of conserved quantities from only specifying the location of a point of interest (POI) that the trajectory is forced to go through (for example, an IBEX position), and the velocity vector at infinity (e.g., core of ISM neutral distribution).
-> Closed, analytic solution available. Specifically also determines velocity at POI, and impact parameter at infinity.

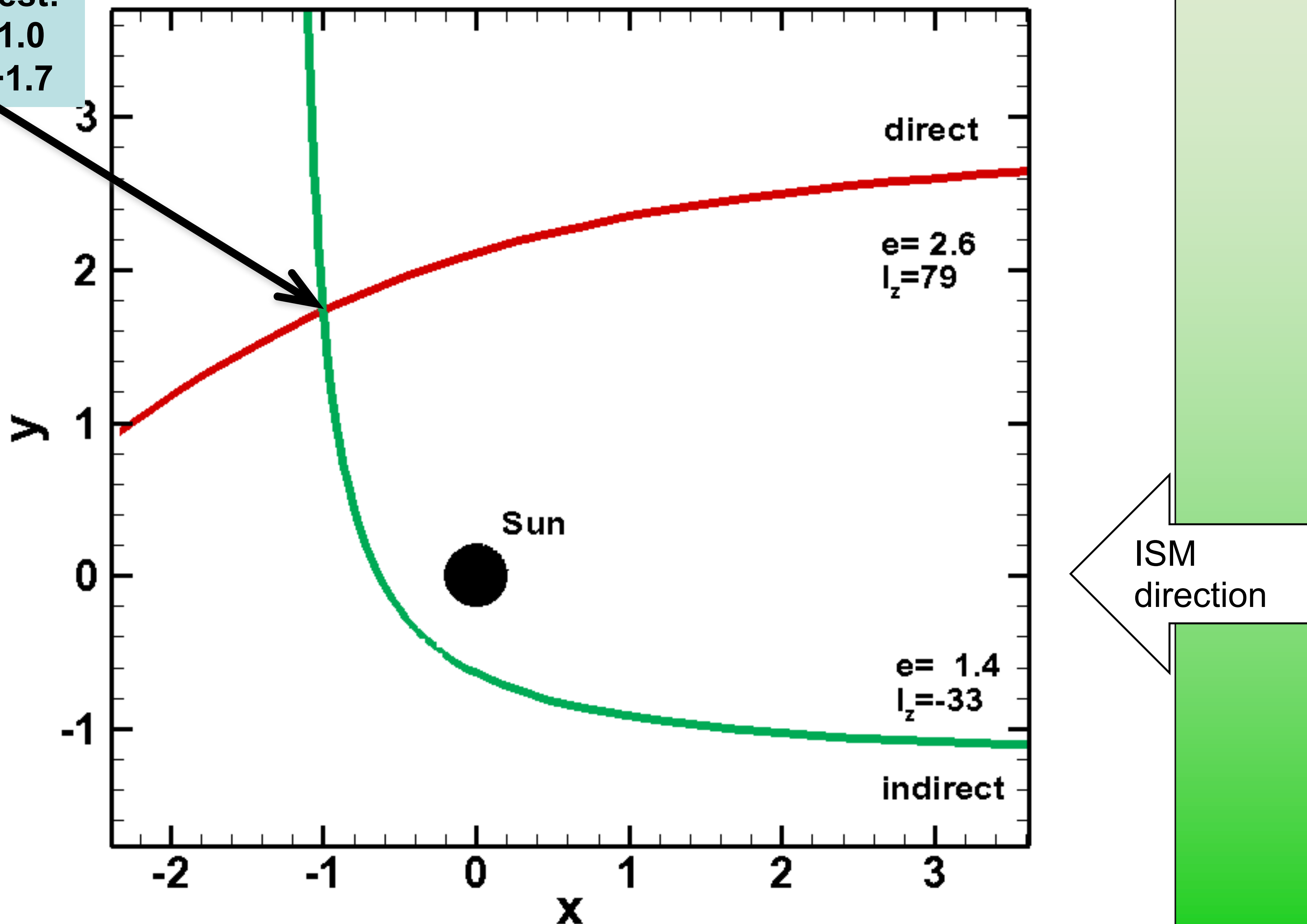
Original/Application frame of reference: An arbitrary, heliocentric inertial frame of reference, for example the Heliocentric Aries Ecliptic (HAE) with x-y plane the ecliptic, and x-axis pointing to a fixed direction in the sky.

ISM frame of reference: inertial frame of reference with positive x-axis pointing antiparallel to the (peak of the) ISM flow so that the ISM velocity vector has components ($u_{x,ism}$<0, 0, 0)

ISM velocity and eccentricity vector define ISM frame and Kepler frame, respectively. All 3 coordinate systems are connected through simple transformation matrices (obtained by compounding rotations).

Direct and indirect paths

Example point of interest:
X = -1.0
Y = +1.7



Calculation method

Type II problem ("backward calculation")
Get a full set of conserved quantities from specifying the location of a point of interest (POI) and the velocity at the POI.
-> Closed, analytic expressions. Specifically also determines velocity at infinity; via the ISM Maxwellian, this velocity translates into a phase space density.

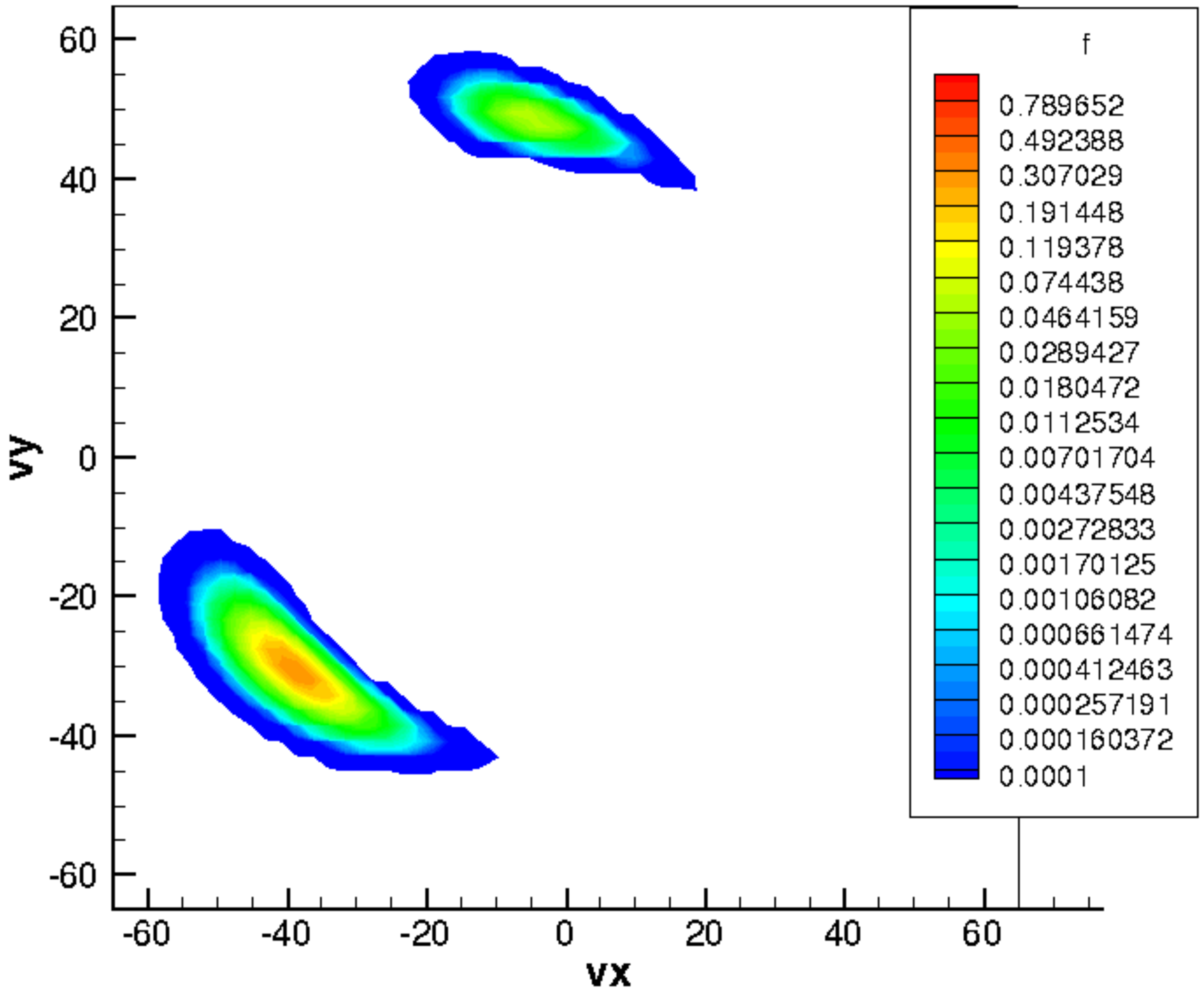
$$f_{ISM}(\vec{r},\vec{v}) = n_{ISM}\left(\frac{m}{2\pi k_B T_{ISM}}\right)^{\frac{3}{2}} \exp\left[-\frac{(v_x - v_{ISM})^2 + v_y^2 + v_z^2}{2k_B T_{ISM}}\right]$$

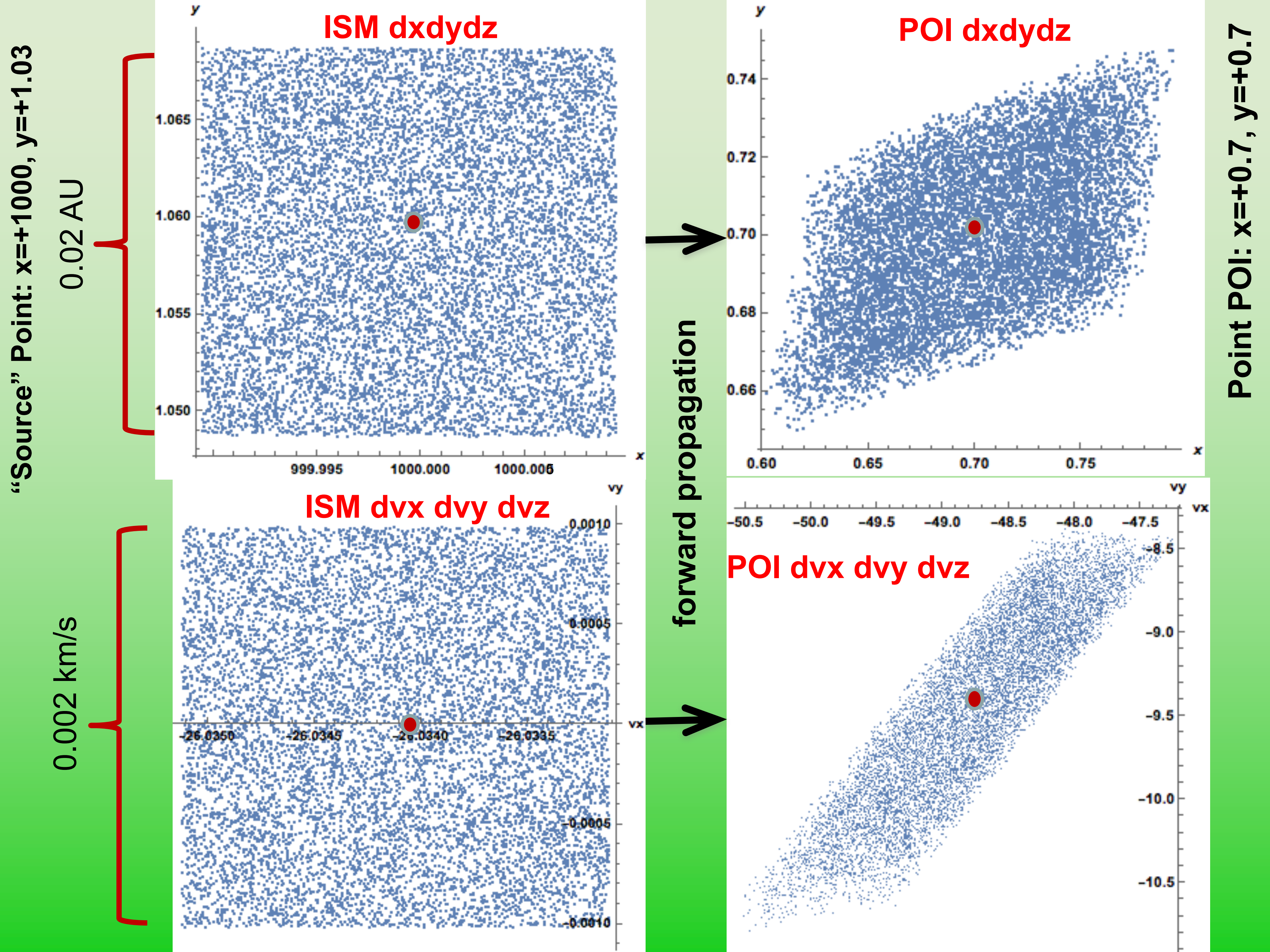
Calculate entire primary VDF: Investigate all velocities ***v*** (in bounding box), and get phase space density via Type II calculation. Scan bounding box as finely as needed. (Current default: 30x30x30 velocities.)

This is equivalent to calculating a family/bundle of trajectories intersecting at the POI, each with a distinct set of conserved quantities and with a distinct Kepler frame of reference (and of course, distinct impact parameters, in other words, neutrals meeting at the POI do not originate from the same point in the ISM).

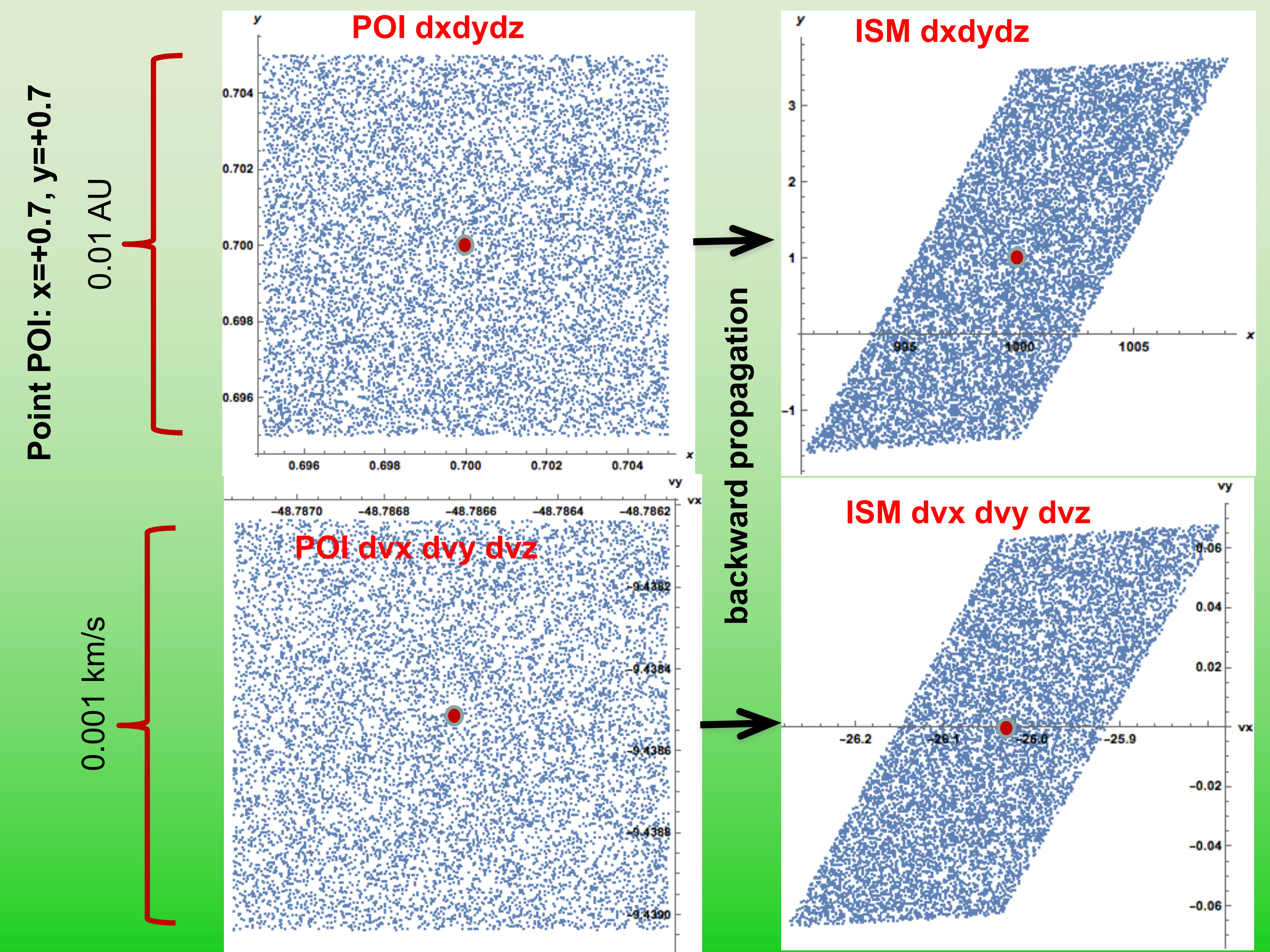
Primary helium VDF

Primary helium (color)
Point: x=-0.7, y=+0.7





"Source" Point: x=+1000, y=+1.03
0.02 AU
ISM dxdydz
0.002 km/s
ISM dvx dvy dvz
forward propagation
POI dxdydz
POI dvx dvy dvz
Point POI: x=+0.7, y=+0.7


Point POI: x=+0.7, y=+0.7
0.01 AU
0.001 km/s
POI dxdydz
ISM dxdydz
POI dvx dvy dvz
ISM dvx dvy dvz
backward propagation
y
x
0.696
0.698
0.700
0.702
0.704
995
1000
1005
vx
vy
-48.7870
-48.7868
-48.7866
-48.7864
-48.7862
-9.4382
-9.4384
-9.4386
-9.4388
-9.4390
-26.2
-26.1
-26.0
-25.9
0.06
0.04
0.02
-0.02
-0.04
-0.06

Going from normalized to absolute Phase Space Density – Liouvill-ian?

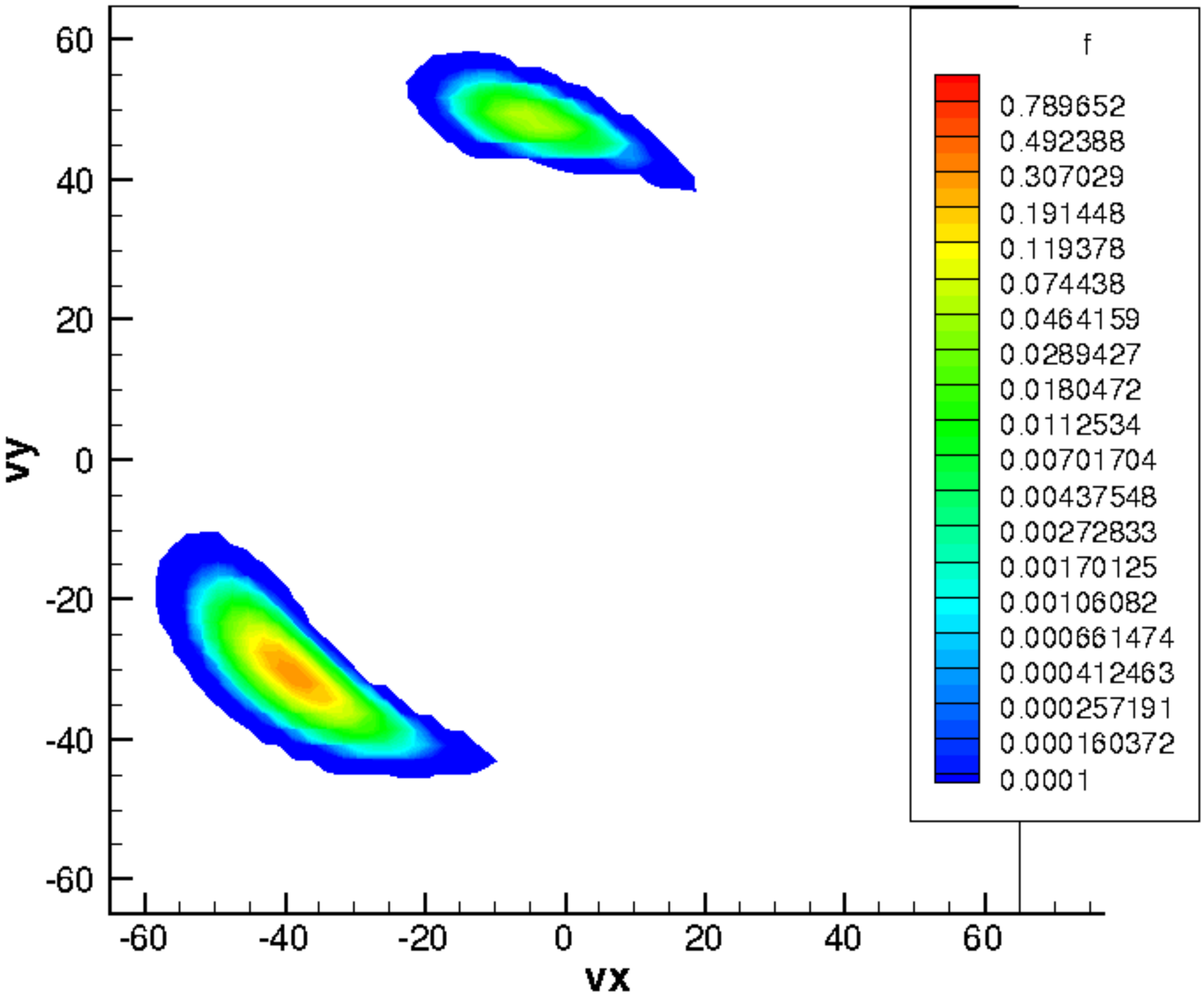
- An ISM point (x_i, y_i, z_i, vx_i, vy_i, vz_i) is mapped one-to-one to a POI (x, y, z, vx, vy, vz) in innermost heliosphere/IBEX/IMAP
- An ensemble of N helium atoms $N = f_{ISM}\, d^3x_{ISM}\, d^3v_{ISM}$ originating at the "ISM source point" lands in the vicinity of the POI, but occupies a much wider range in space and in velocity space.
- (Liouville says, however, that the six-dimensional volume does not change, and remains $d^3x_{ISM}\, d^3v_{ISM}$; it's just impossible to visualize this properly – too many dimensions. See wikipedia for a x – vx example [2D phase space])

CONVERSELY: (backward direction)

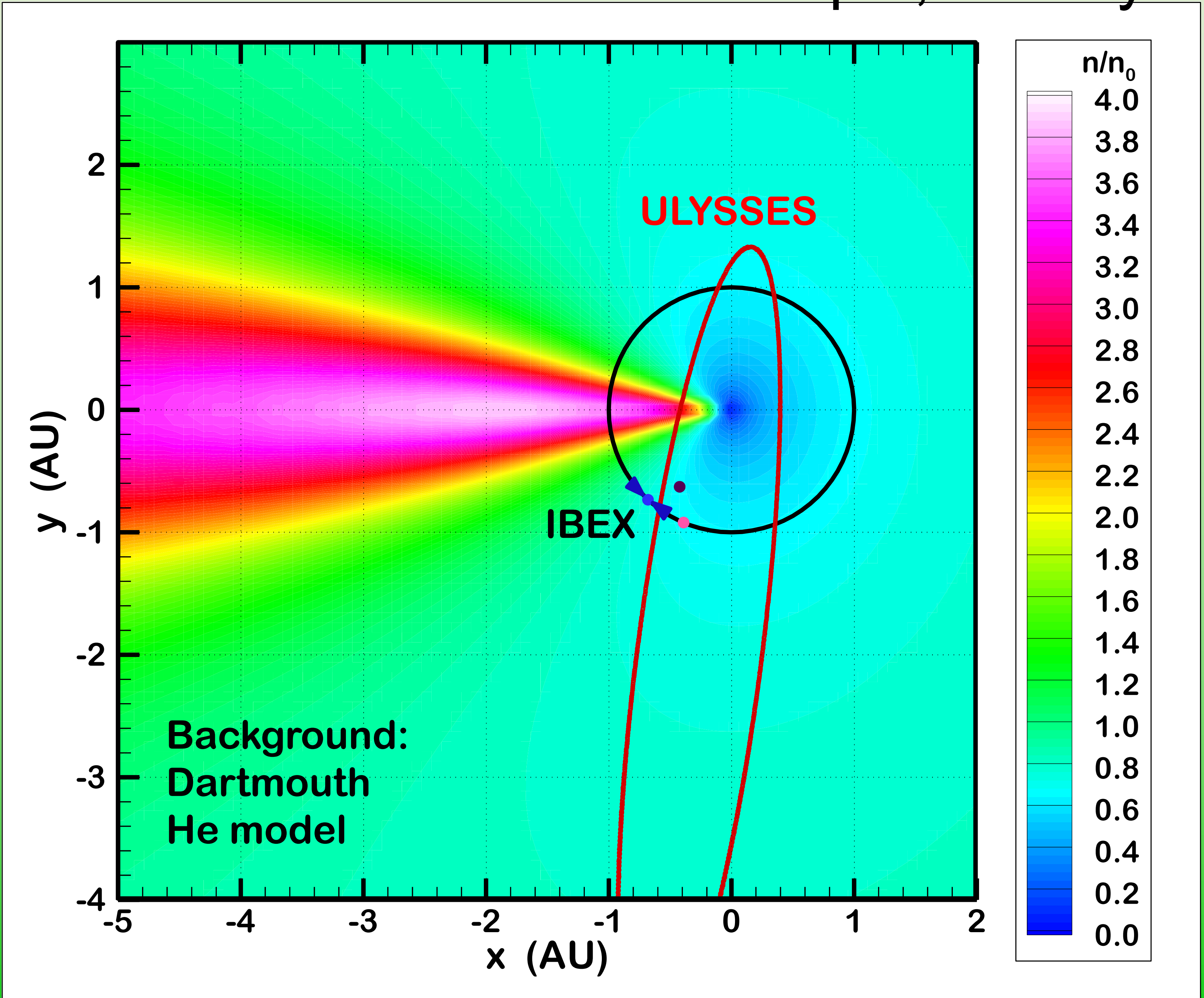
- A POI (x, y, z, vx, vy, vz) in innermost heliosphere/IBEX/IMAP is mapped one-to-one to an ISM point (x_i, y_i, z_i, vx_i, vy_i, vz_i)
- The phase space density at POI gets contributions ***not only*** from the "ISM source point", but lots of others! Assigning a PSD value to a POI may not be as simple as getting the PSD value at the corresponding ISM point at infinity, but likely needs to be the result of an integration over the source map that corresponds to a $d^3x\, d^3v$ at the POI.

Primary helium (color)
Point: x=-0.7, y=+0.7

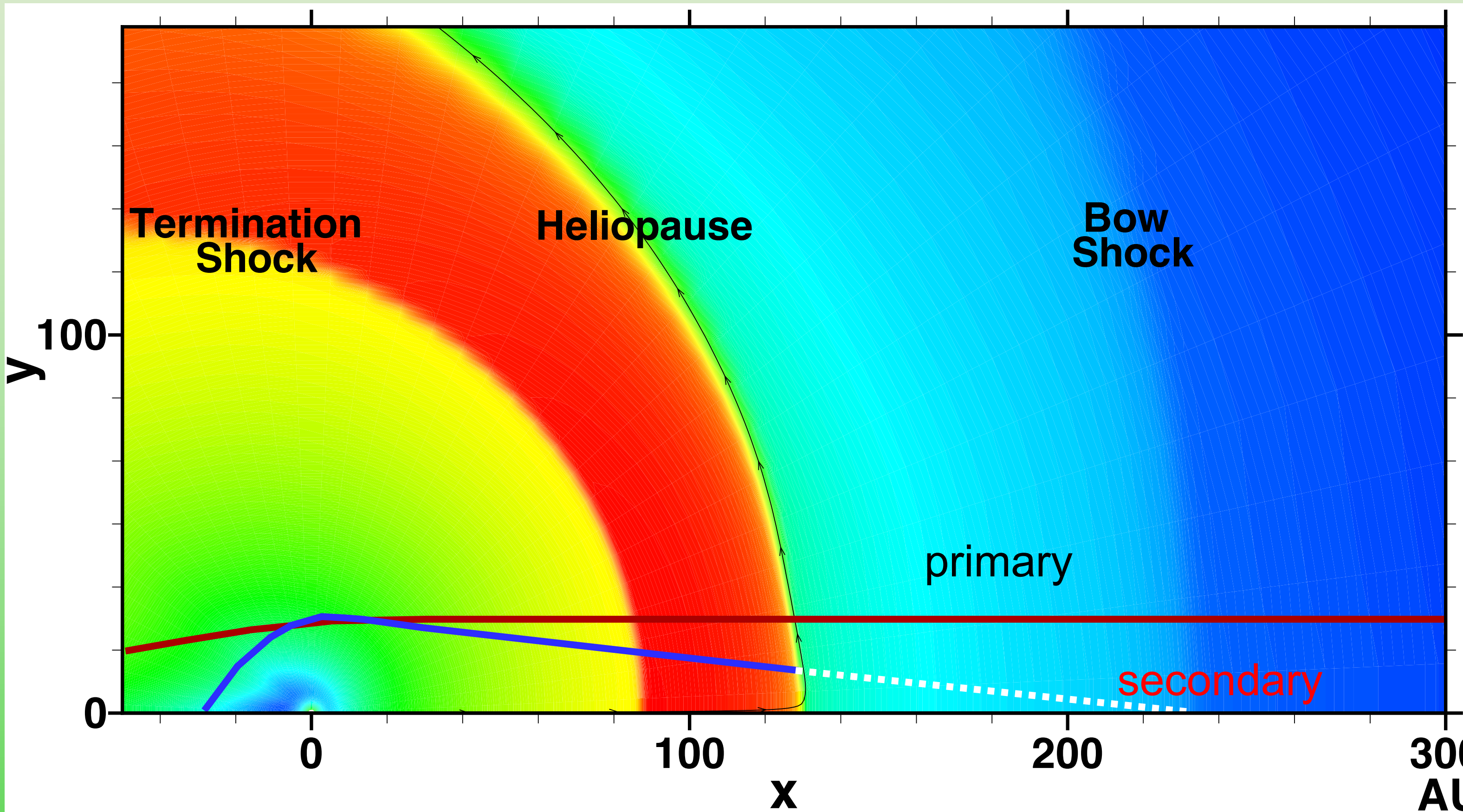
Primary helium VDF



Moments of VDF: For example, density



Secondary production



Exaggerated schematic view of primary and secondary pathways to a point in the inner heliosphere

Secondary neutral helium: Distributions and Sources

Describe movement of **primary and secondary neutral He atoms** as a **Kepler trajectory** – analytically known

Couple many such trajectories to a global heliosphere model; treat He atoms as **test particles**

Together, can calculate local neutral He **loss**
and local neutral He **production**

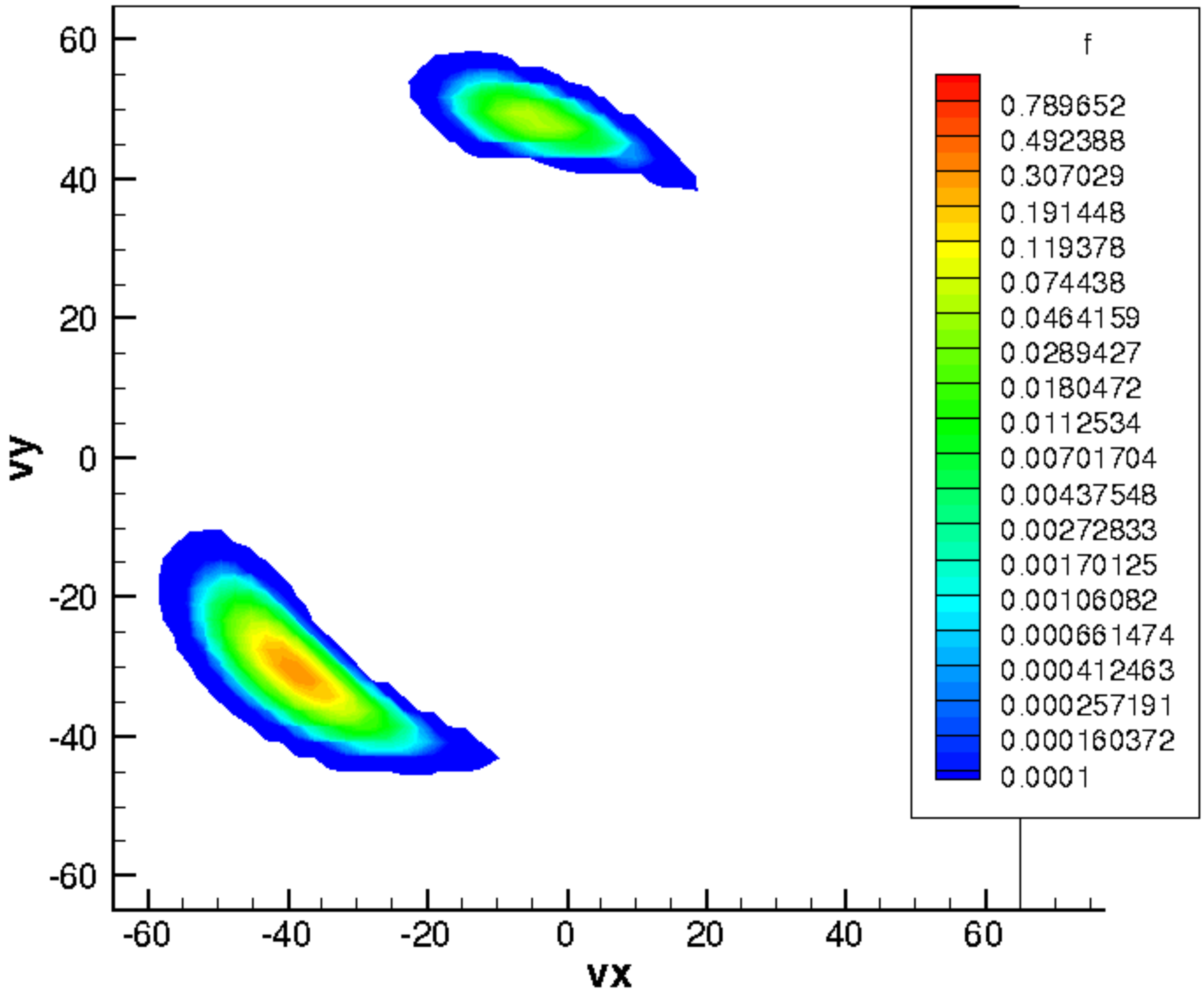
Can **integrate** losses, leading to “survival probability”, “optical depth”.
Together with interstellar source of primary He, leads to
filtered primary He fluxes at IBEX

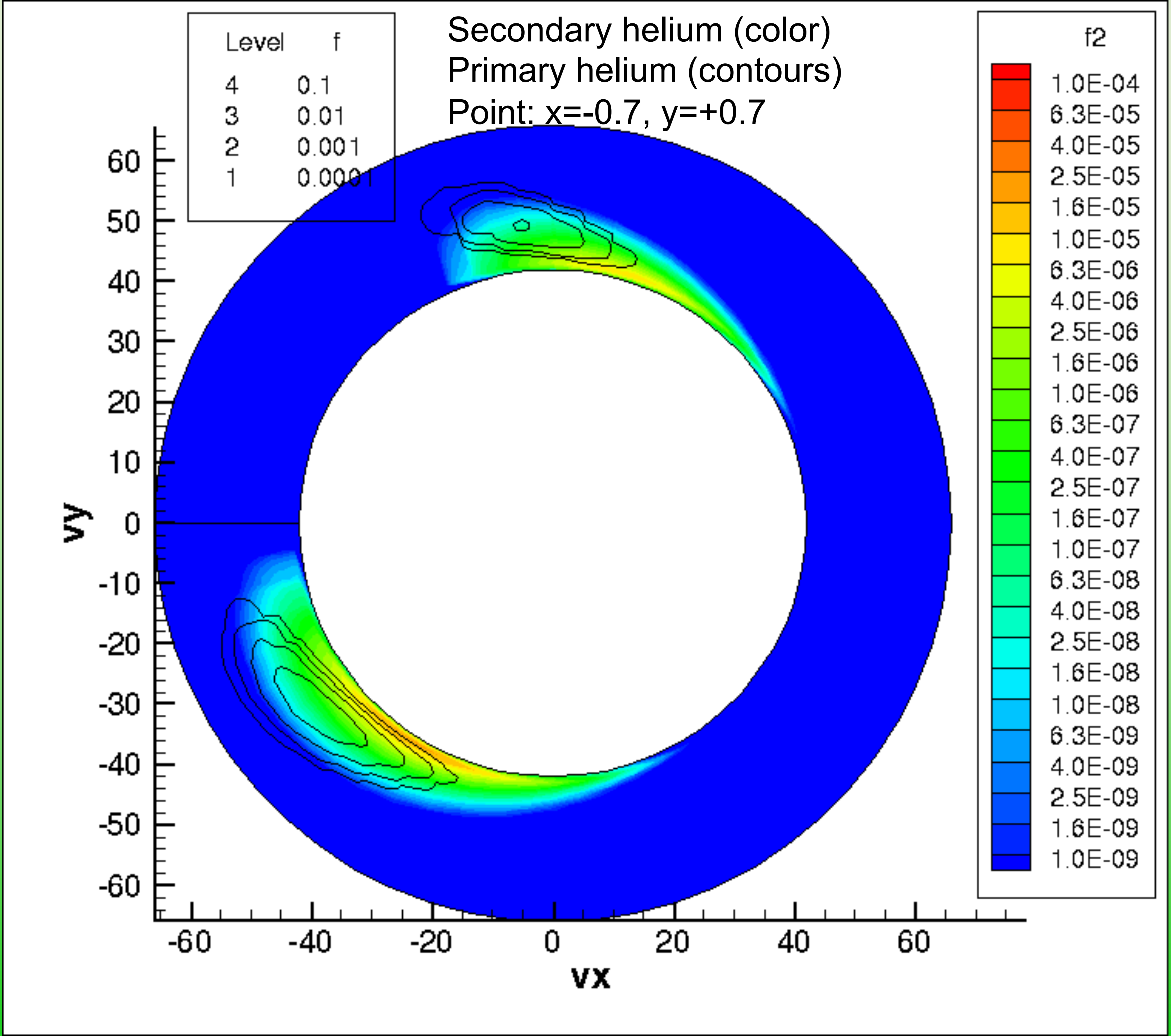
Can integrate production along the trajectory, leading to
(**filtered) secondary He fluxes** at IBEX

Instead of fluxes at IBEX, will concentrate here on **Velocity Distribution Functions** (VDF) in heliocentric inertial frame of reference

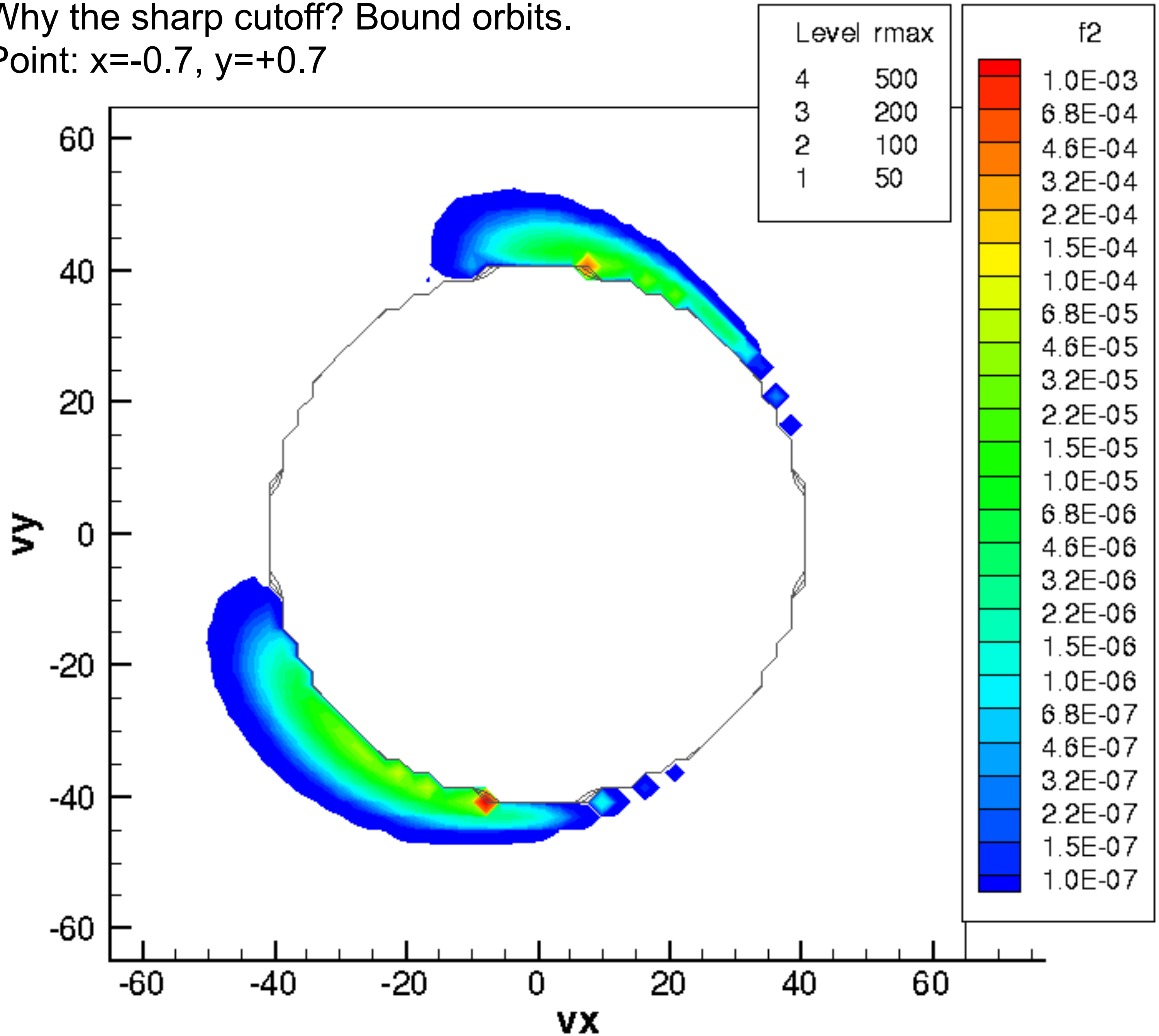
Primary helium (color)
Point: x=-0.7, y=+0.7

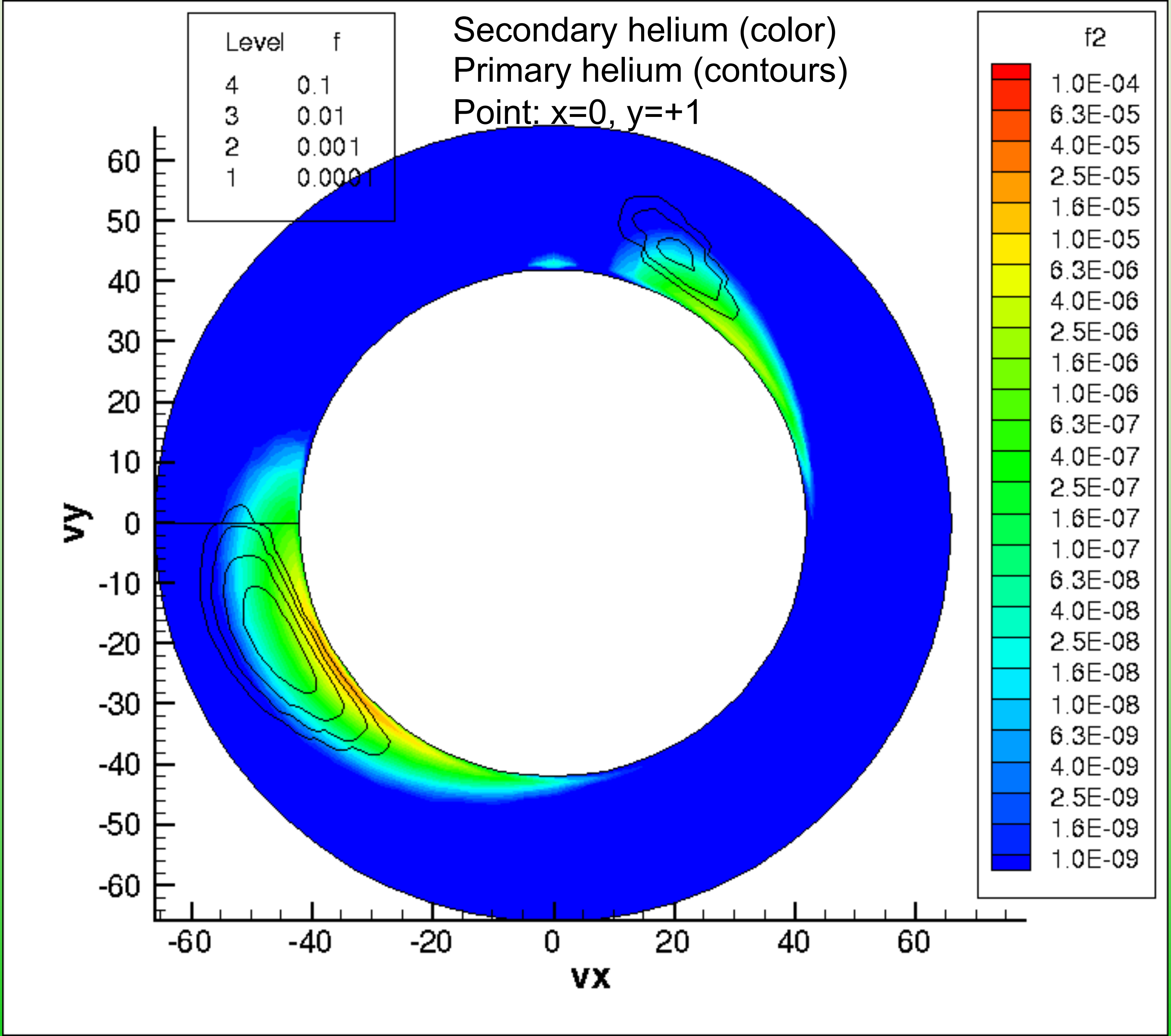
Primary helium VDF

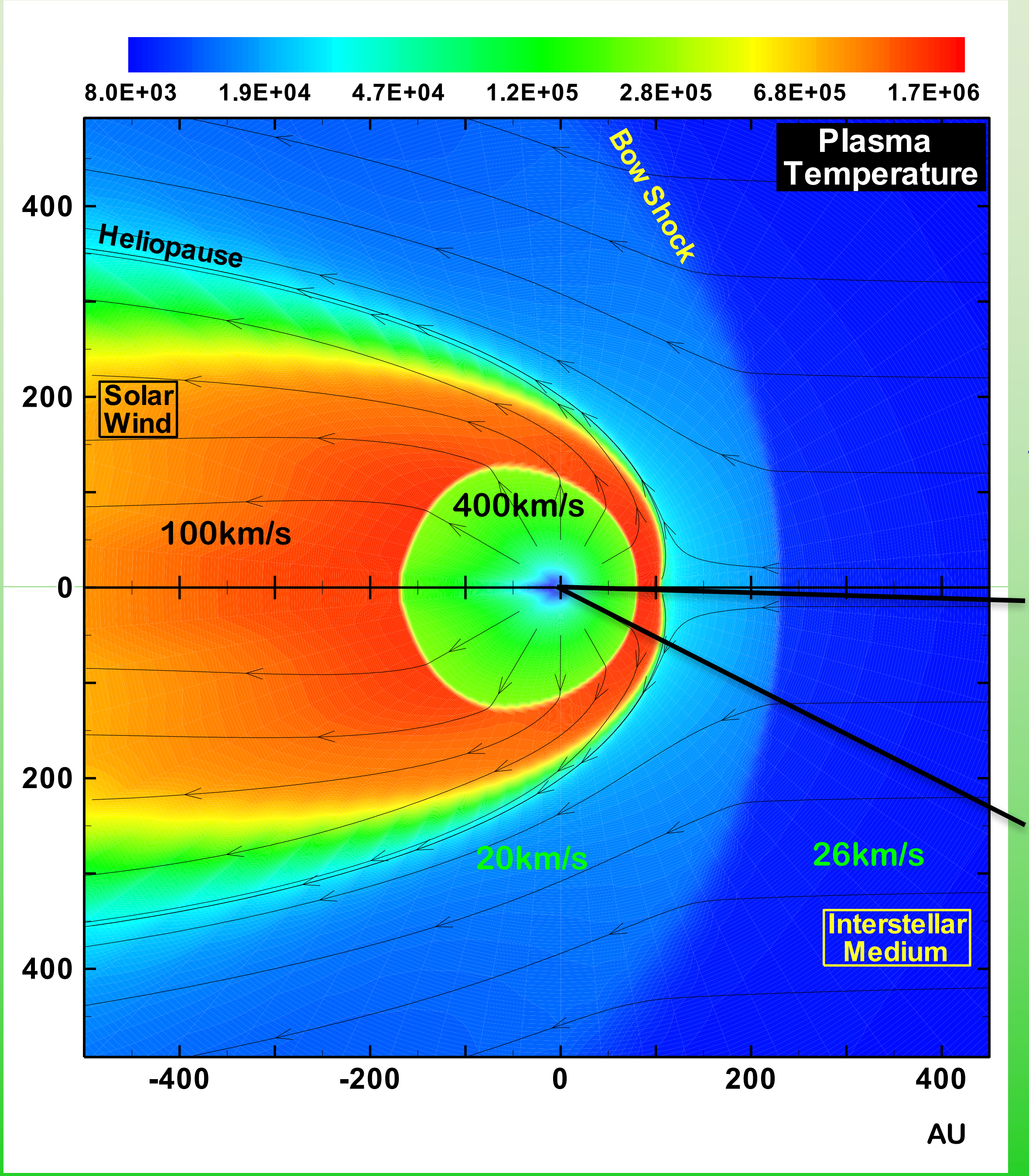



Secondary helium (color)
Primary helium (contours)
Point: x=-0.7, y=+0.7
Level f
4 0.1
3 0.01
2 0.001
1 0.0001
f2
1.0E-04
6.3E-05
4.0E-05
2.5E-05
1.6E-05
1.0E-05
6.3E-06
4.0E-06
2.5E-06
1.6E-06
1.0E-06
6.3E-07
4.0E-07
2.5E-07
1.6E-07
1.0E-07
6.3E-08
4.0E-08
2.5E-08
1.6E-08
1.0E-08
6.3E-09
4.0E-09
2.5E-09
1.6E-09
1.0E-09
vy
vx

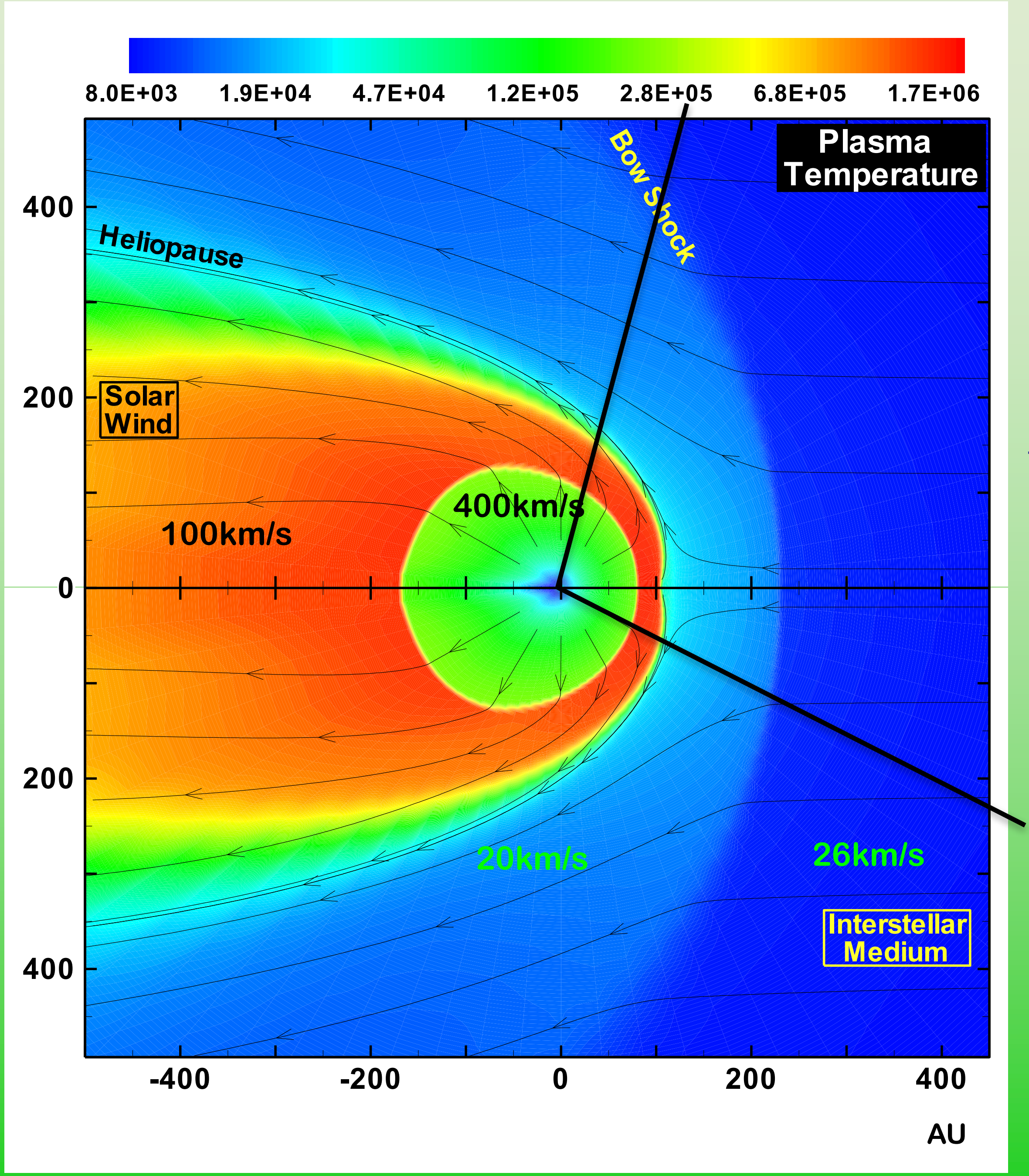
Why the sharp cutoff? Bound orbits.
Point: x=-0.7, y=+0.7




Level f
4 0.1
3 0.01
2 0.001
1 0.0001
Secondary helium (color)
Primary helium (contours)
Point: x=0, y=+1
f2
1.0E-04
6.3E-05
4.0E-05
2.5E-05
1.6E-05
1.0E-05
6.3E-06
4.0E-06
2.5E-06
1.6E-06
1.0E-06
6.3E-07
4.0E-07
2.5E-07
1.6E-07
1.0E-07
6.3E-08
4.0E-08
2.5E-08
1.6E-08
1.0E-08
6.3E-09
4.0E-09
2.5E-09
1.6E-09
1.0E-09
vy
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
vx
-60
-40
-20
0
20
40
60



Secondary Helium –– particle trajectory leading to direct peak



Secondary Helium –– particle trajectory leading to indirect peak

Conclusions

- Primary helium at various IBEX and other spacecraft locations can be calculated “ab-initio” using an accurate method
- Secondary helium at various IBEX and other spacecraft locations can be calculated “ab-initio” using an accurate method, but involve an integration and the involvement of a global heliosphere model that represents the He^+ ions (interstellar origin) in the outer heliosheath upwind of the heliopause.
- Sample calculations show promise that secondaries contribute to IBEX-Lo ISN signal, the Warm Breeze, etc
- The issues surrounding the Liouville argument need to be worked out to check for subtle asymmetries.