Turbulence Analysis of a Flux Rope Plasma on the Swarthmore Spheromak Experiment

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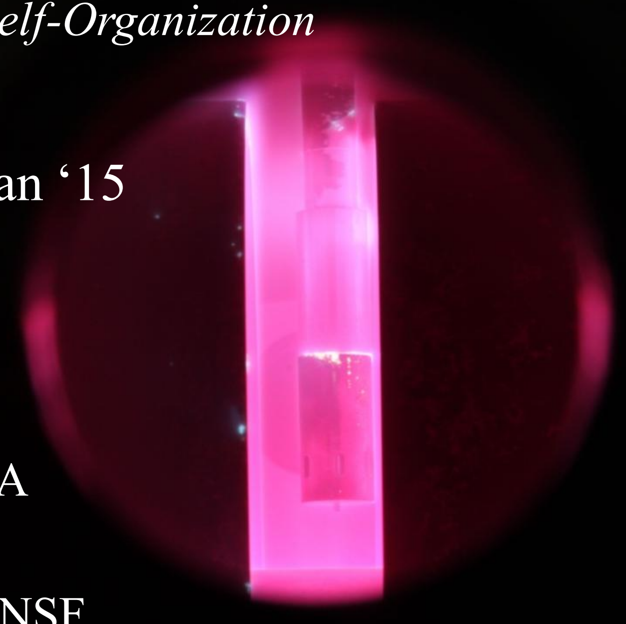
with contributions from
M. Brown, V. Lukin, T. Gray, A. Wan '15



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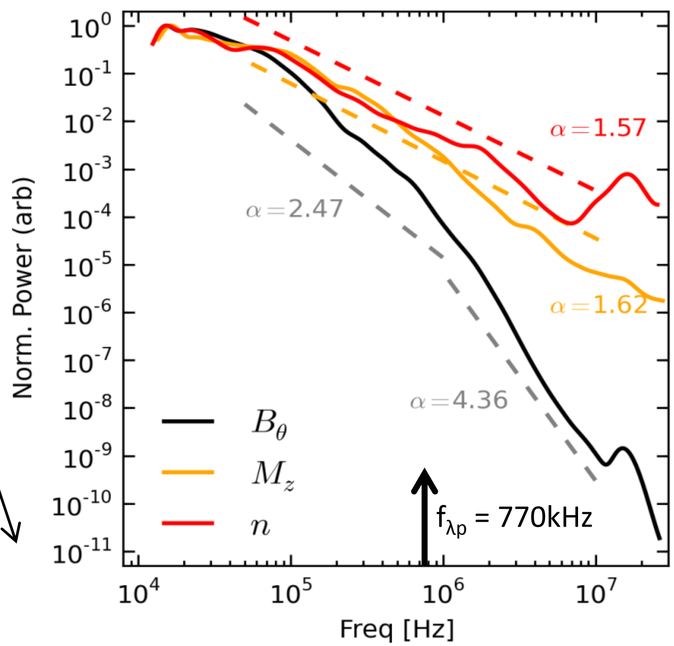
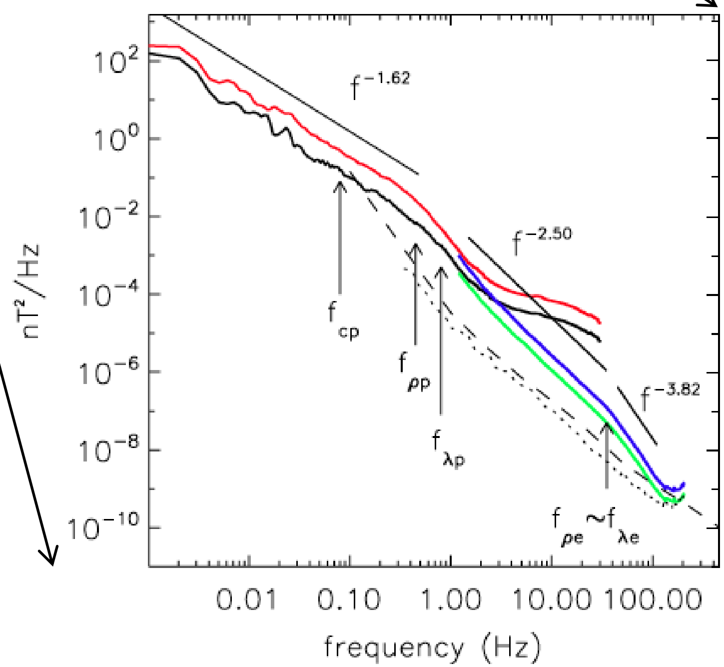
Research supported by US DOE and NSF

Is the statistical character of MHD turbulence universal?

Laboratory: SSX



Space: Solar Wind*



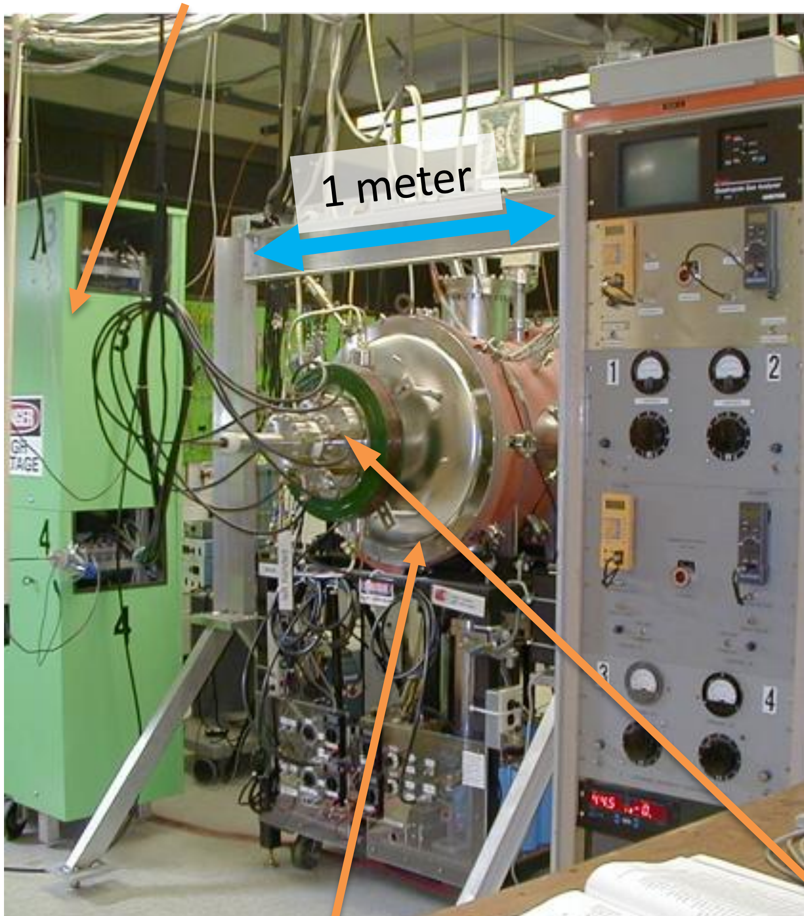
*Cluster FGM and STAFF-SC Data: *Sahraoui, PRL 2009*

Goals for this Talk

1) Give overview of SSX plasma:
   a) Formation
   b) Selective Decay (Magnetic Self-Organization)
2) Reveal turbulent characteristics of SSX
   a) Power spectrum
   b) PDF of increments
3) Show injected helicity scan
   a) Affect of structure size on turbulence

The SSX Laboratory

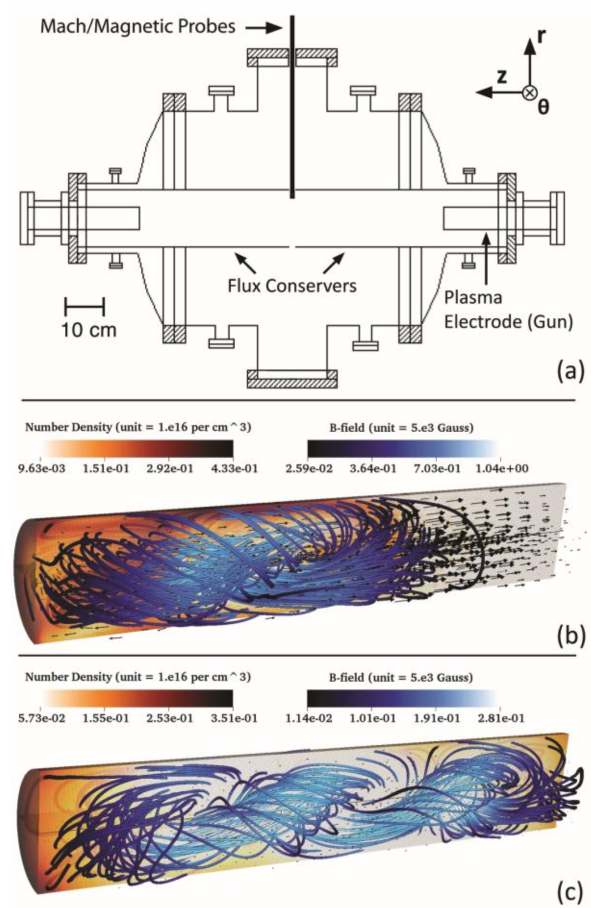
10kV/100kA Pulsed power 1mF banks



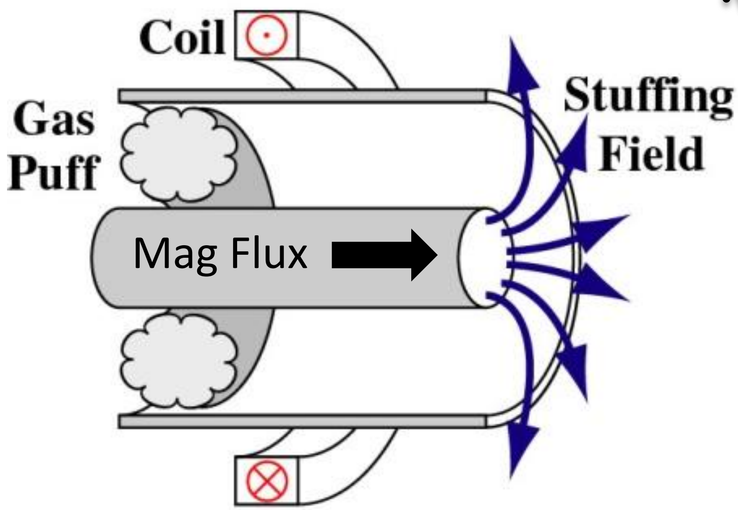
Cylindrical vacuum chamber (D = 0.5 m, L = 1 m)

High voltage plasma guns on each end

Schematic + Simulation



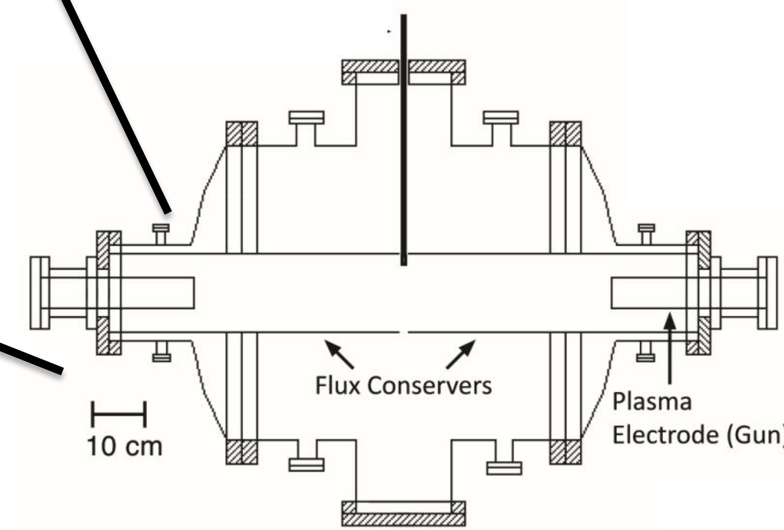
Spheromak Formation w/Plasma Gun



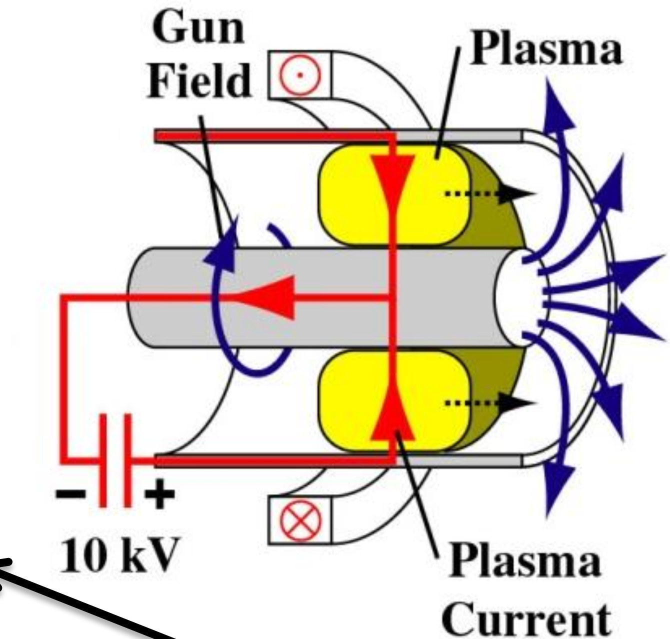
Step 1:

a) Produce Stuffing Field
b) Introduce Hydrogen gas

Zoom



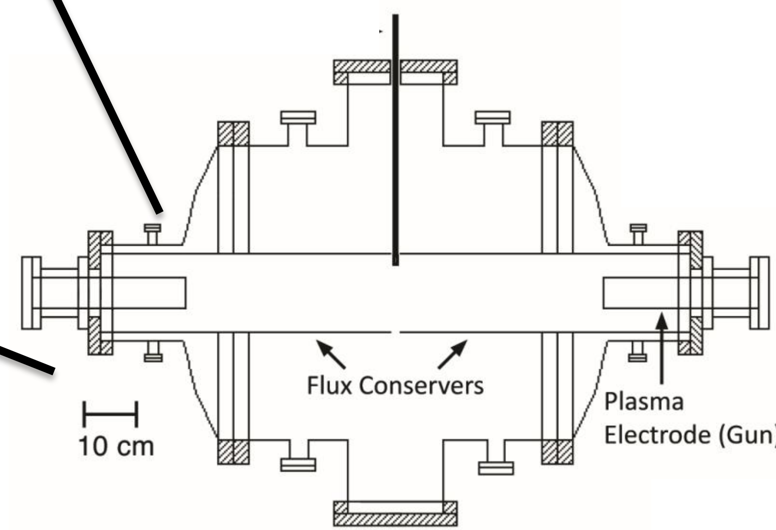
Spheromak Formation w/Plasma Gun



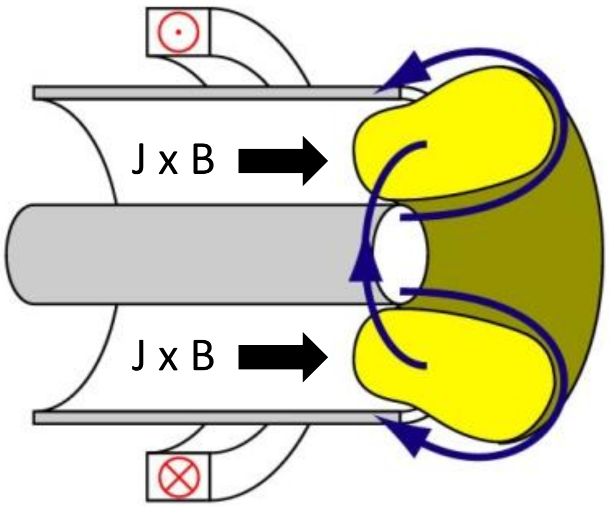
Step 2:

a) Apply bias across gun
b) Produce plasma

Zoom

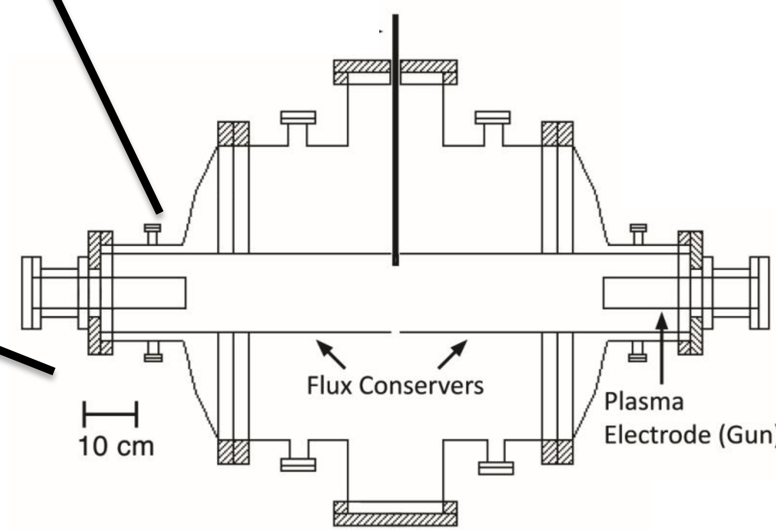


Spheromak Formation w/Plasma Gun

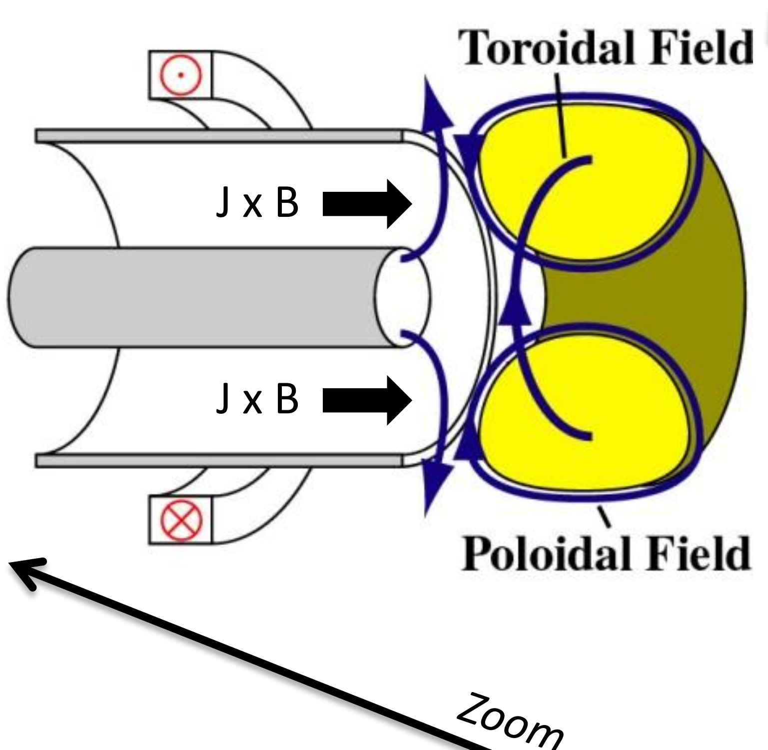


Step 3:

a) J x B force pushes plasma out of gun
b) Spheromak begins to form

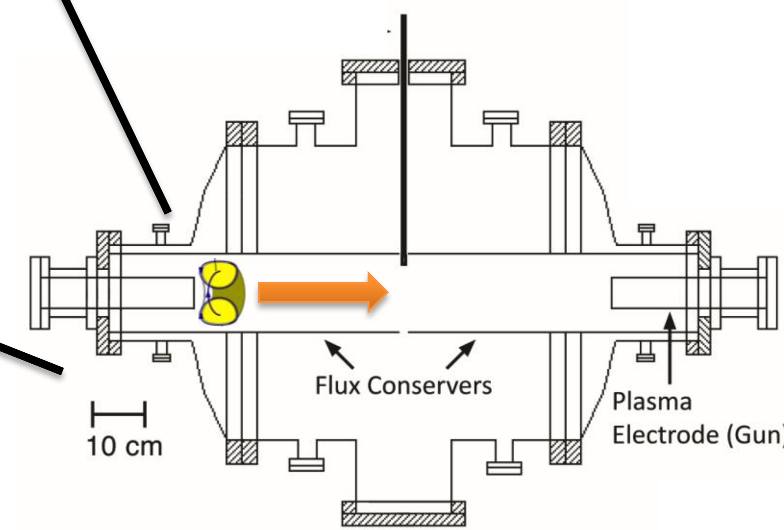


Spheromak Formation w/Plasma Gun

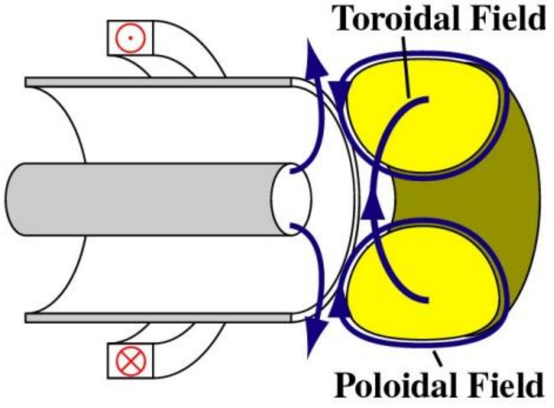


Step 3:

a) Spheromak breaks off from gun, pushed into chamber



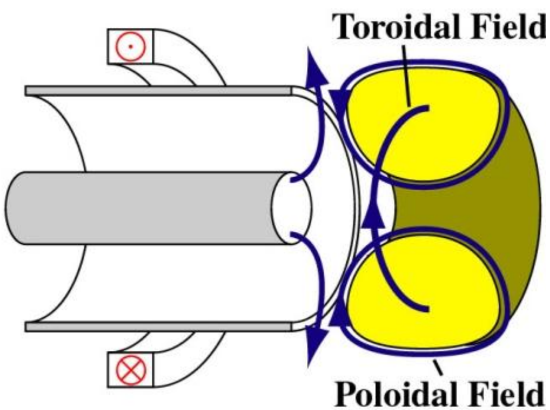
To Tilt or Not to Tilt?

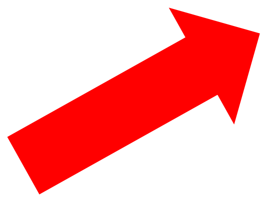


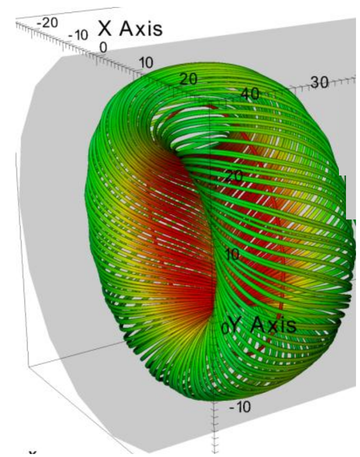
One of two things happened next: dependant on boundary

To Tilt or Not to Tilt?

Small Aspect Ratio
(Wide Chamber):

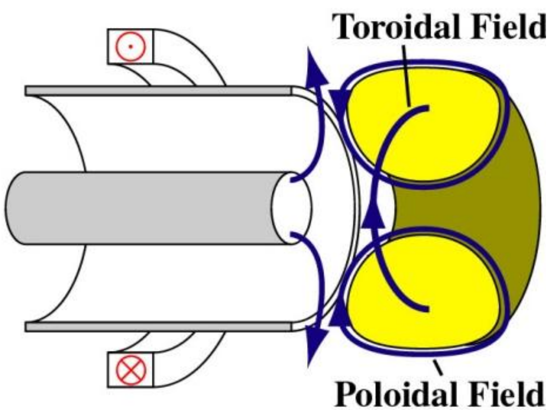






Stable Spheromak
Toroidal Shape

To Tilt or Not to Tilt?



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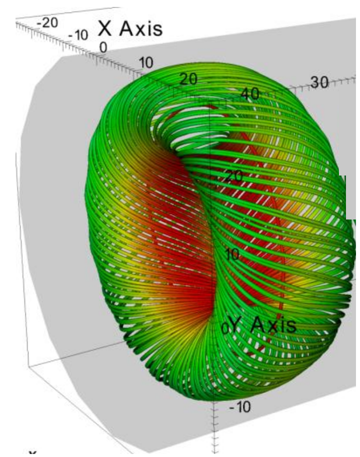
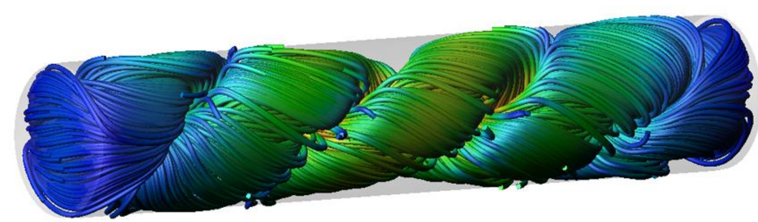


Stable Spheromak
Toroidal Shape

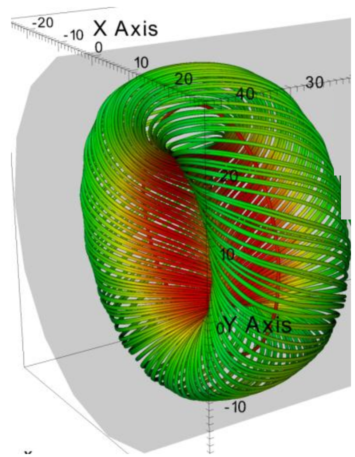
Large Aspect Ratio
(Narrow Chamber):



Taylor State (Tilt and Twist)
Double Helix Shape

To Tilt or Not to Tilt?

Small Aspect Ratio
(Wide Chamber):



Stable Spheromak
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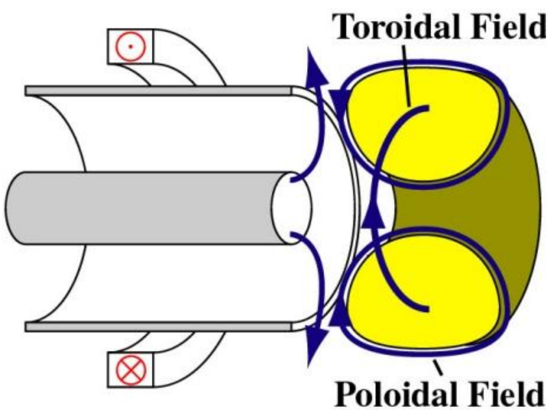

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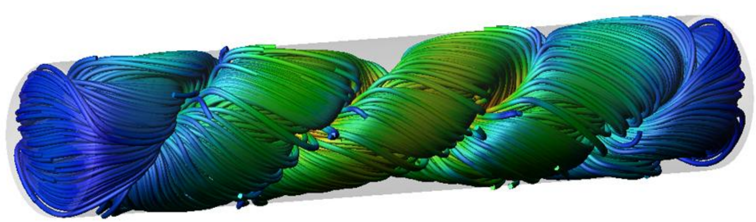


Taylor State (Tilt and Twist)
Double Helix Shape

Taylor relaxation (selective decay) provides energy injection for MHD turbulence cascade

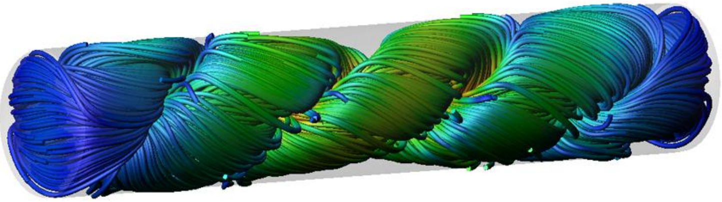
Taylor relaxation or selective decay of a plasma is a process where magnetic energy,

$$E_B = \int \frac{B^2}{2\mu_0} dV$$

is minimized under the constraint that magnetic helicity,

$$K_B = \int A \cdot B dV$$

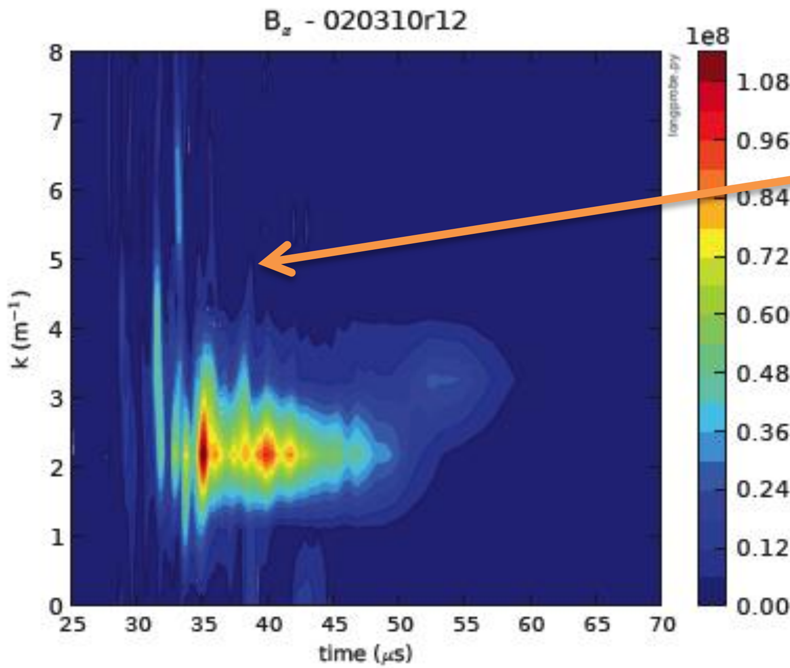
is conserved.



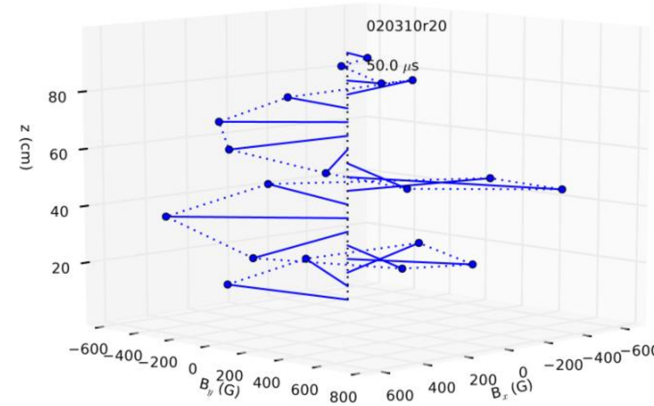
A double helix is the result of this process for a cylindrical boundary
Originally predicted by J. B. Taylor
(above image is analytical calculation)

Previous work* showed that selective decay is observed in the SSX wind tunnel configuration

Bx



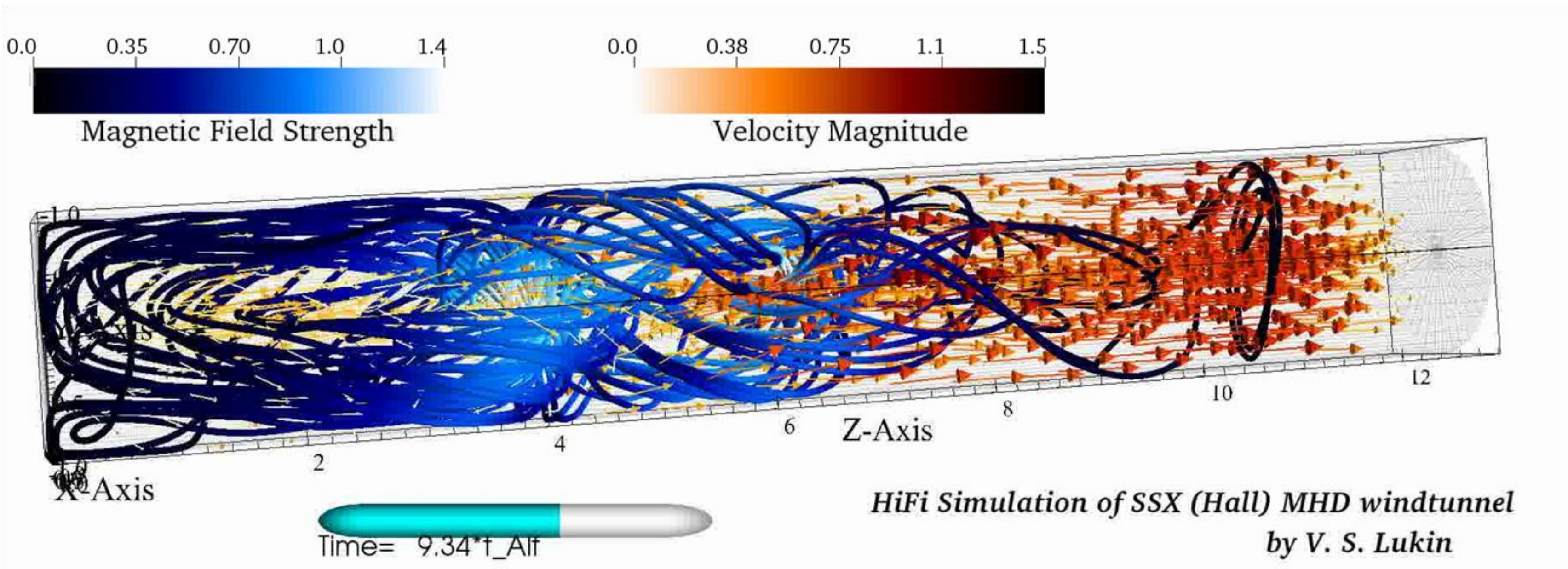
Broadly distributed energy in wavenumber space collapses to large scales in ~ 1 Alfven crossing time



Measured fields achieve helical pitch

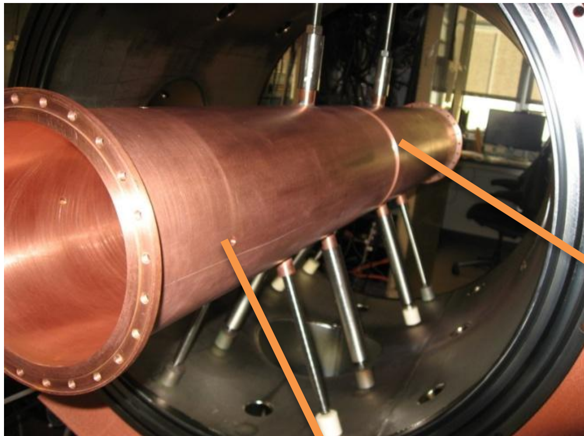
*Gray PRL 2013

Hall MHD simulation clearly illustrates selective decay process



SSX Diagnostics measure B-field, density, flow, soft X-ray light, ion temperature, and gun I/V

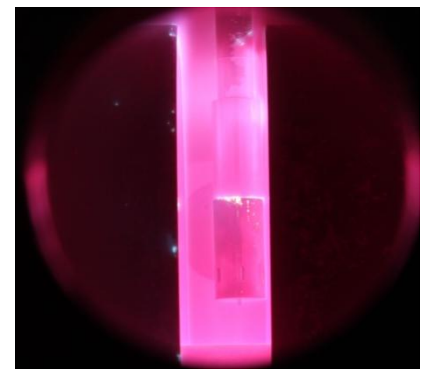
Copper Flux Conserver (Wind Tunnel)



At Midplane:

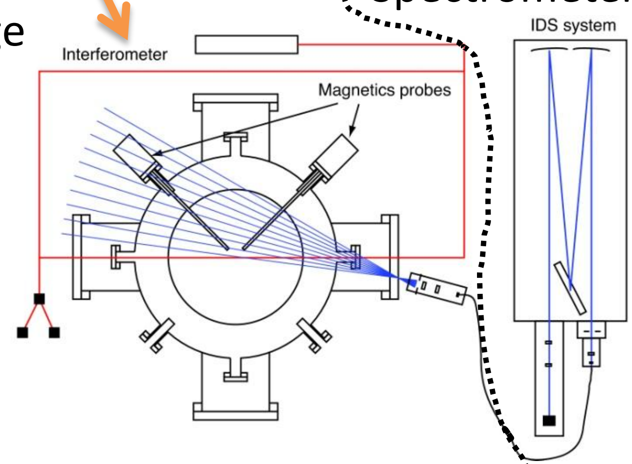
16chan, 3 axis
Bdot Probe

2-Side (Z-axis)
Mach Probe

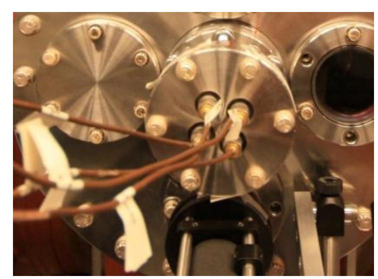


Off Midplane:
HeNe Interferometer
Gun Current/Voltage

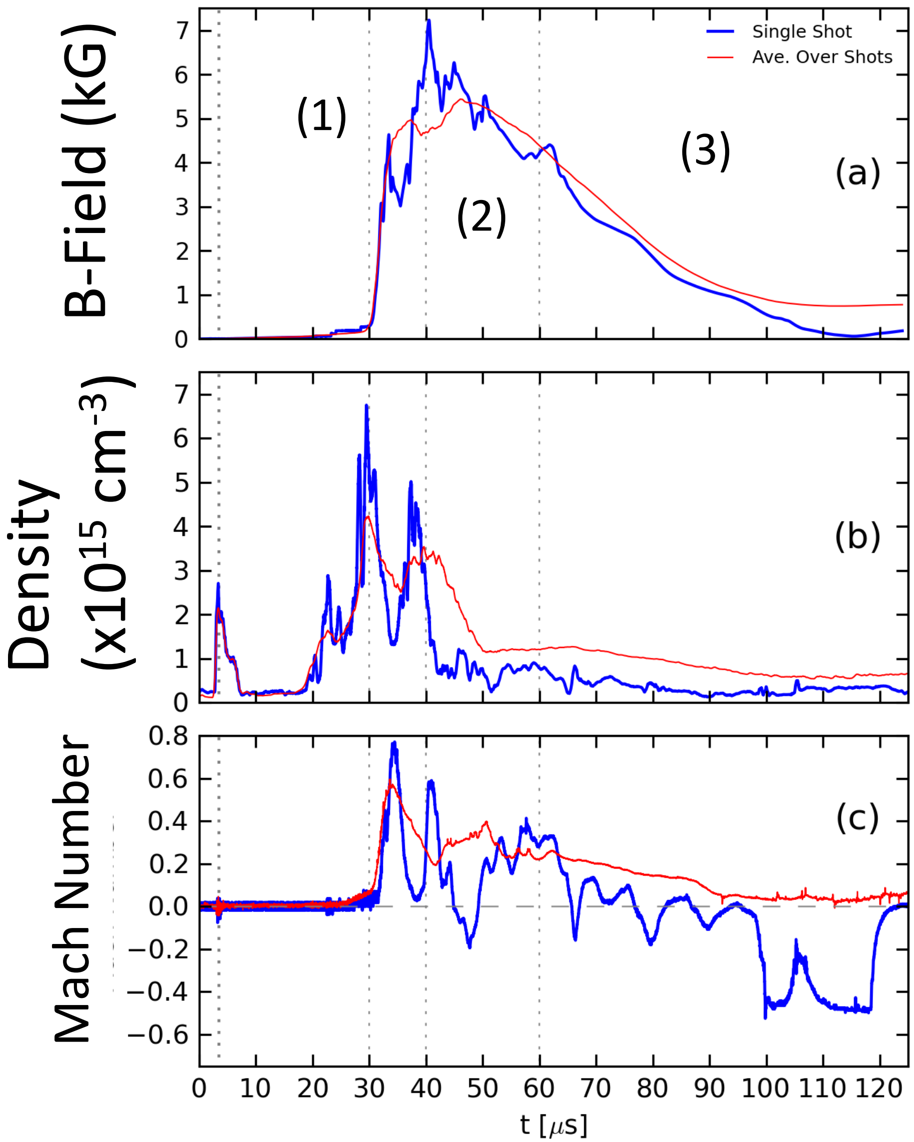
Ion Doppler
Spectrometer (IDS)



Al/Zr Metal Soft X-ray
Photodiode



Magnetized plasma flux robe observed



Three epochs:

1) Formation (selective decay)
2) **Equilibrium**
3) Dissipation (of magnetic structure)

Turbulent characteristics analyzed in equilibrium epoch (40-60us after discharge)

Primary turbulence analysis techniques

Power spectrum

$$E(\omega) = \left[\int B(t) e^{-i\omega t} dt \right]^2$$

Indicates energy transfer rate from scale to scale

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Autocorrelation

$$C(\tau) = \langle \dot{B}(t) \dot{B}(t+\tau) \rangle$$

Indicates temporal decorrelation of fluctuations

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$$R_{ij}(x) = \langle \dot{B}_i(r) \dot{B}_j(r+x) \rangle$$

Indicates spatial decorrelation of fluctuations

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Probability Distribution Function of Increments

$$\Delta \mathbf{b}(t, \Delta t) = \mathbf{b}(t + \Delta t) - \mathbf{b}(t)$$
$$S^2(\Delta t) = \langle (\mathbf{b}(t + \Delta t) - \mathbf{b}(t))^2 \rangle$$

Indicates level of intermittancy in timeseries

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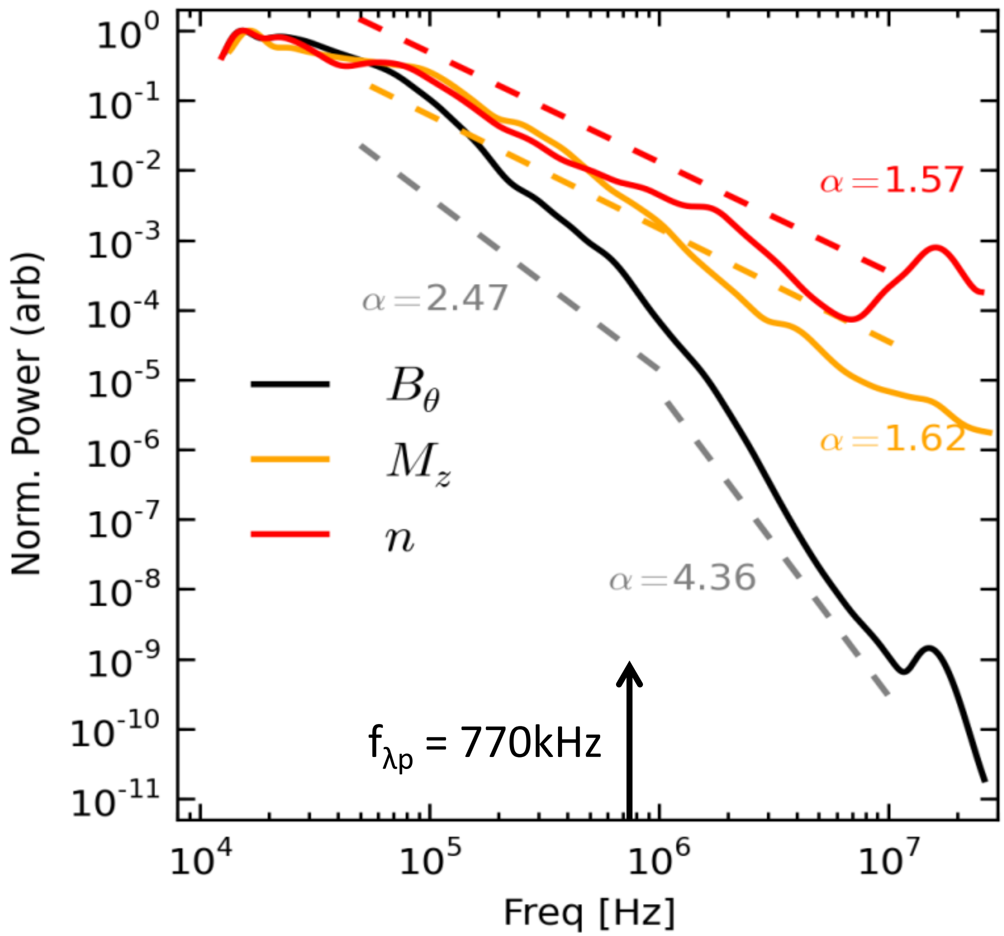
$$S^2(\Delta t) = \langle(\mathbf{b}(t+\Delta t) - \mathbf{b}(t))^2\rangle$$

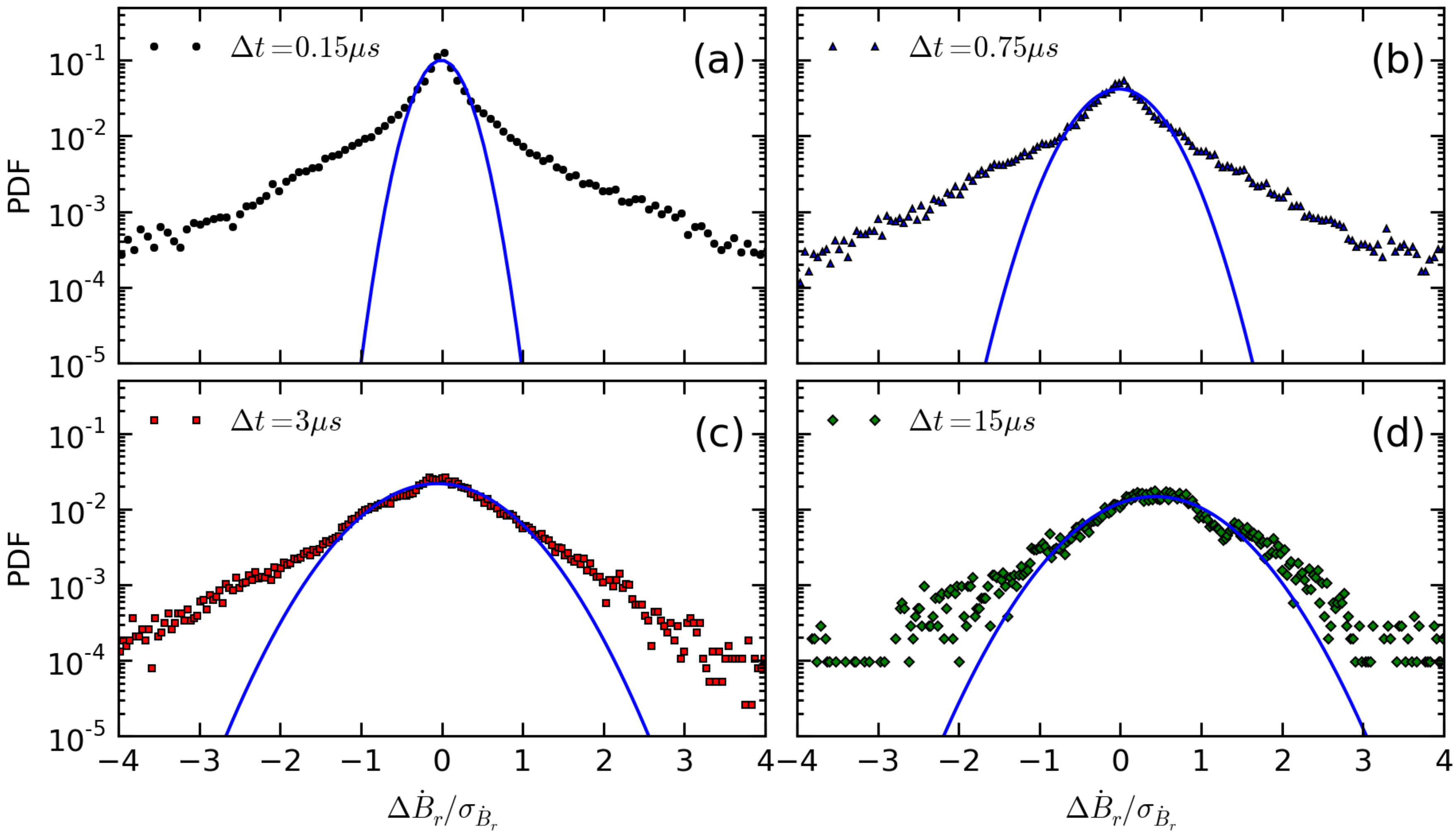
Indicates level of intermittancy in timeseries

Energy spectrum indicates power-law behavior



- B-field spectrum shows power-law behavior in two regions
  - Break perhaps due to dissipation physics--occurs around Doppler shifted ion inertial length frequency, $f_{\lambda p}$
- Flow/Density spectra show power-law behavior with a smaller spectral index
- For reference, Kolmogorov scaling: α= 5/3 = 1.66

PDF of increments have fat tails (intermittency): shows turbulence is highly structured



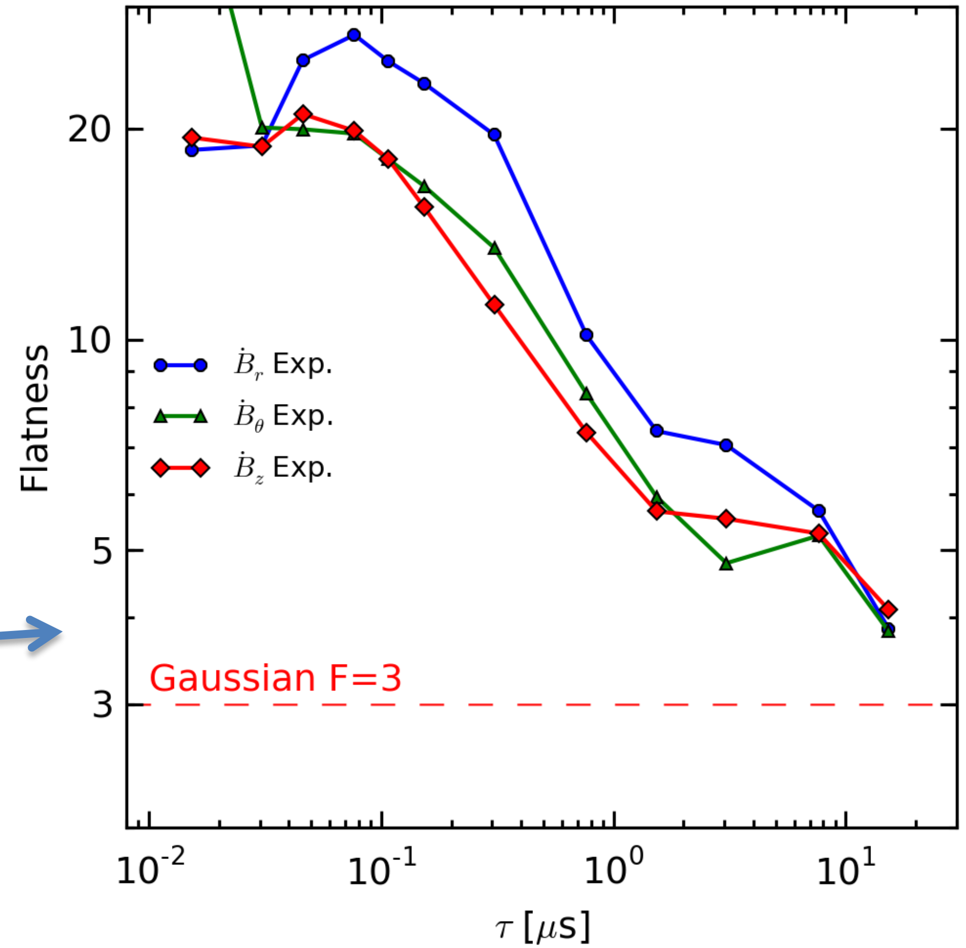
Turbulence theory predicts power-law relationship between flatness of PDF vs size of timestep

Flatness = normalized 4th moment of a PDF

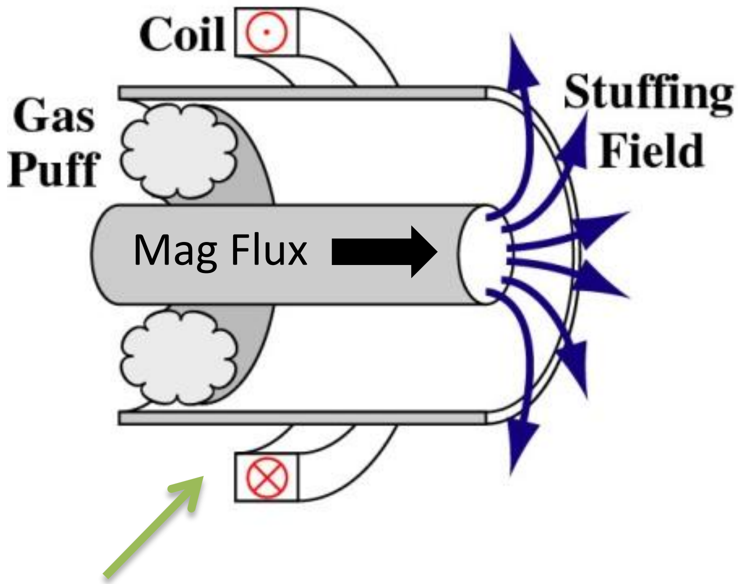
$$F = \frac{\int x^4 P(x)}{(\int x^2 P(x))^2}$$

Flatness = quantification of fat tails, departure from Gaussian → the larger the flatness, the more intermittency is observed

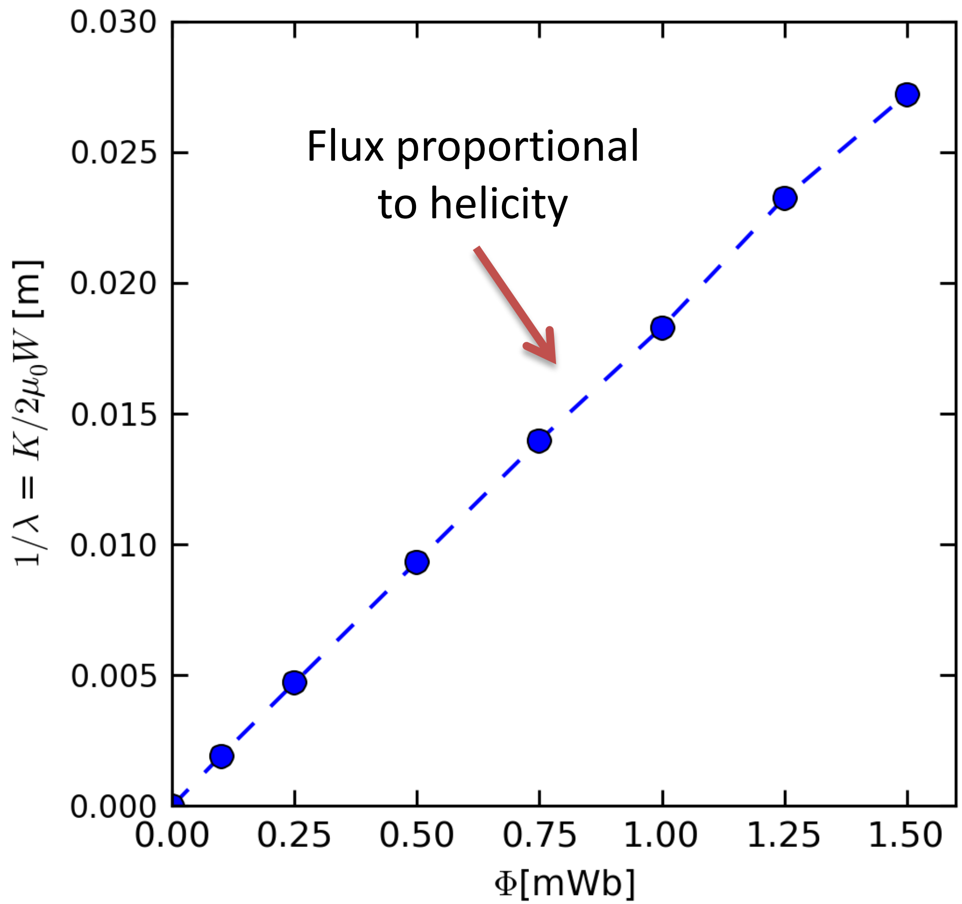
Power-law like scaling is observed in experiment (log-log plot)



Utilizing laboratory plasmas—vary injected helicity



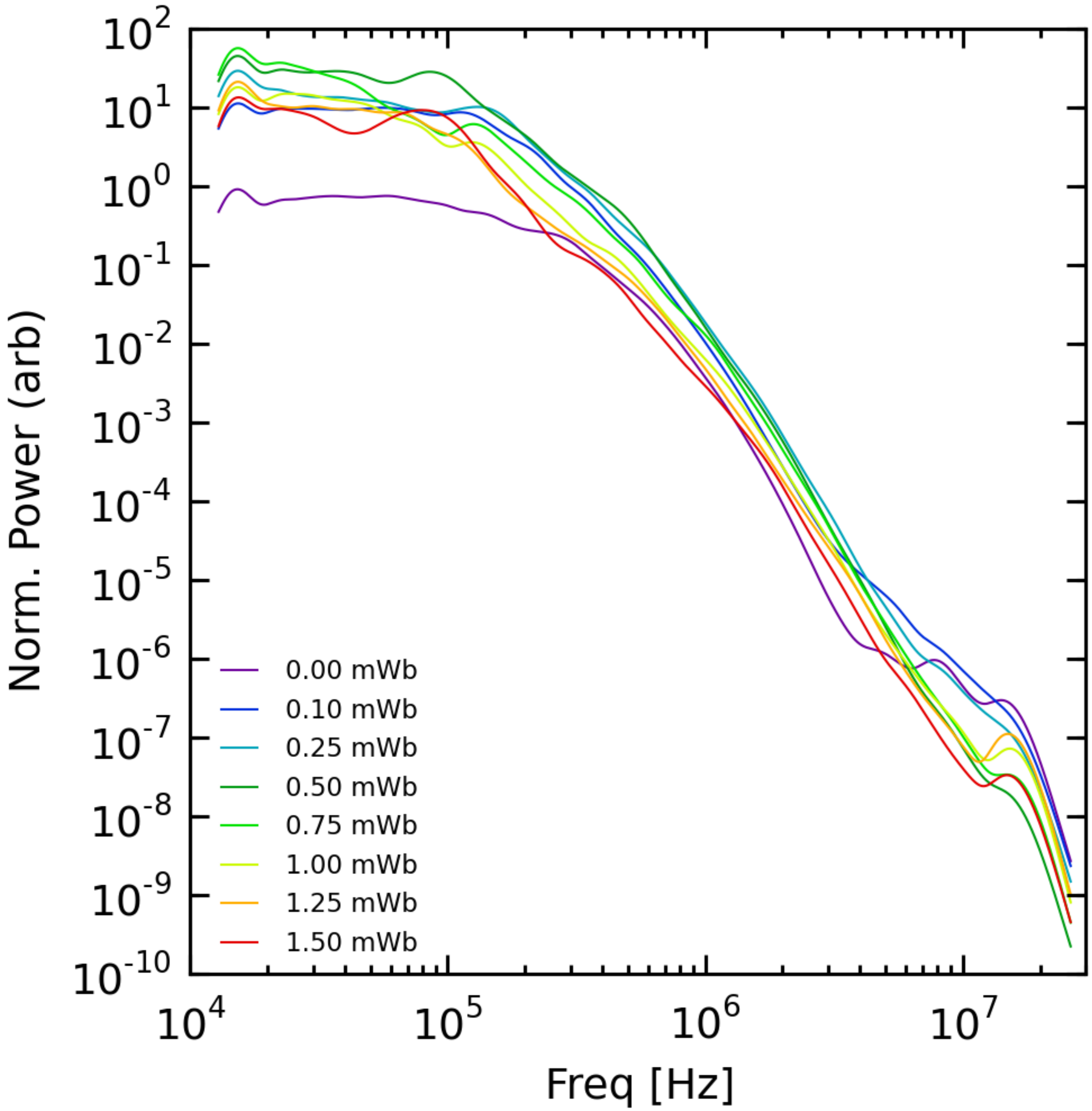
Vary flux by varying strength of field produced by coil



Normalize helicty to injected energy,
W → has units of length scale

$$\frac{1}{\lambda} = \frac{K}{2\mu_0 W} = \frac{1}{\mu_0} \frac{\int \Phi V_{gun} dt}{\int I_{gun} V_{gun} dt}$$

Shape/slopes of power spectrum generally unchanged



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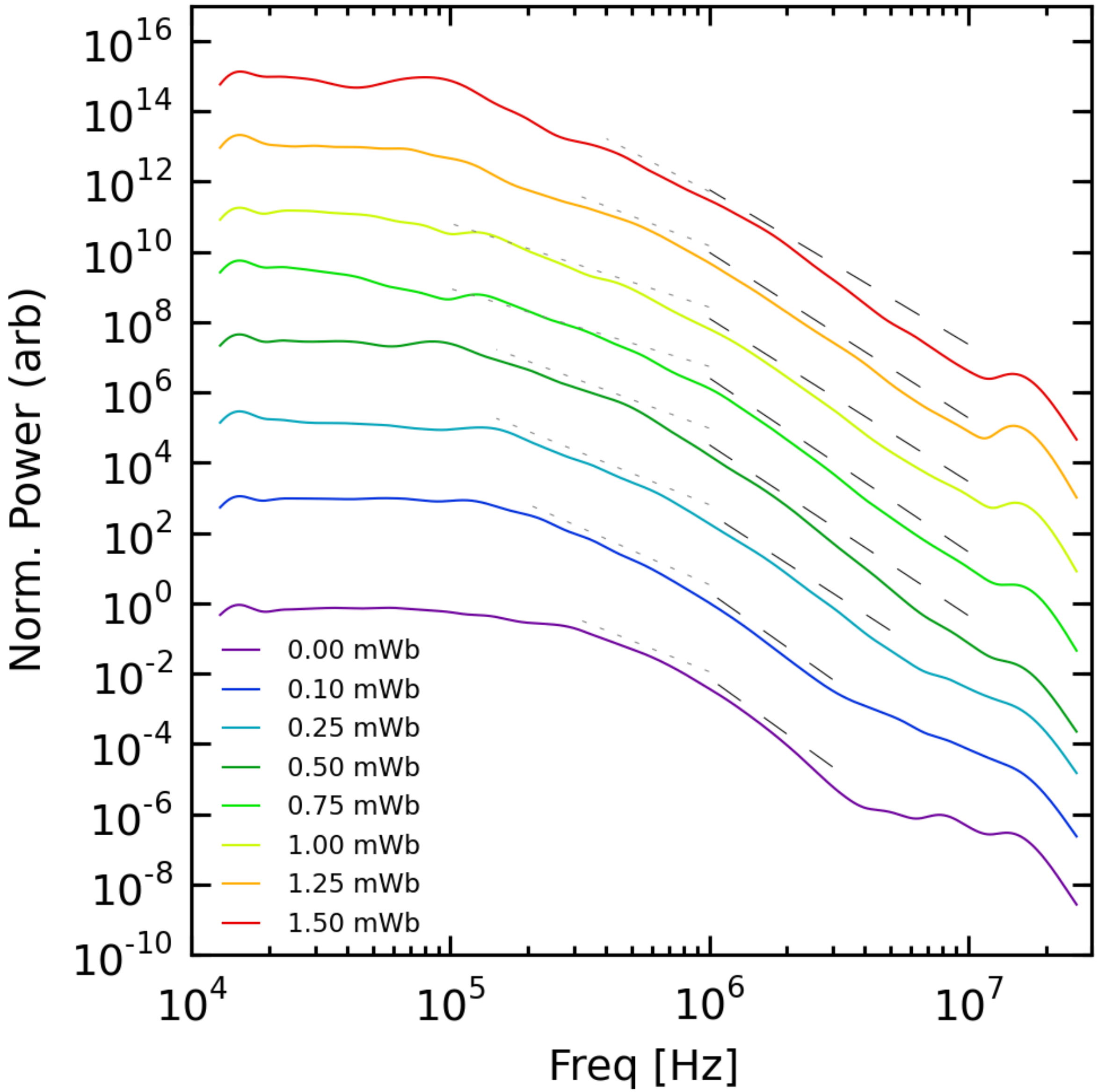
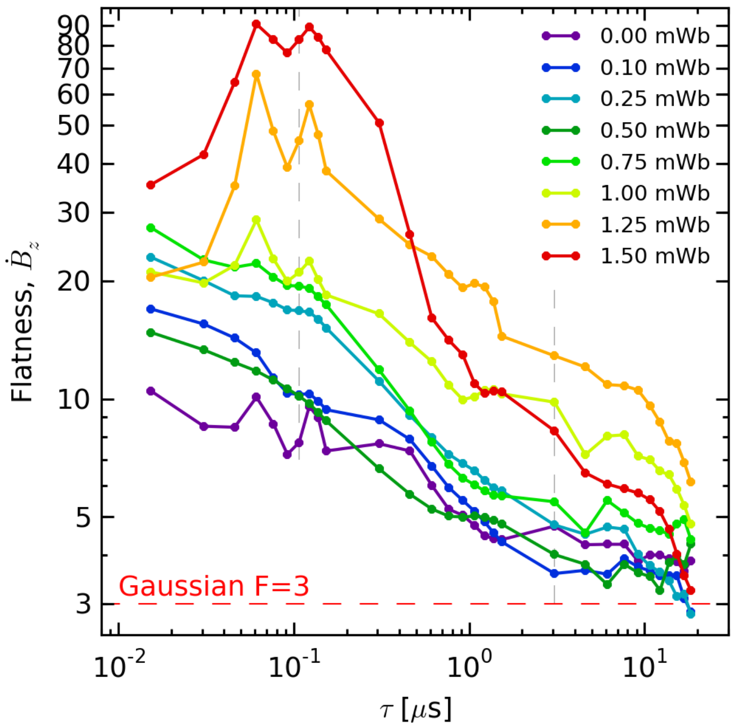
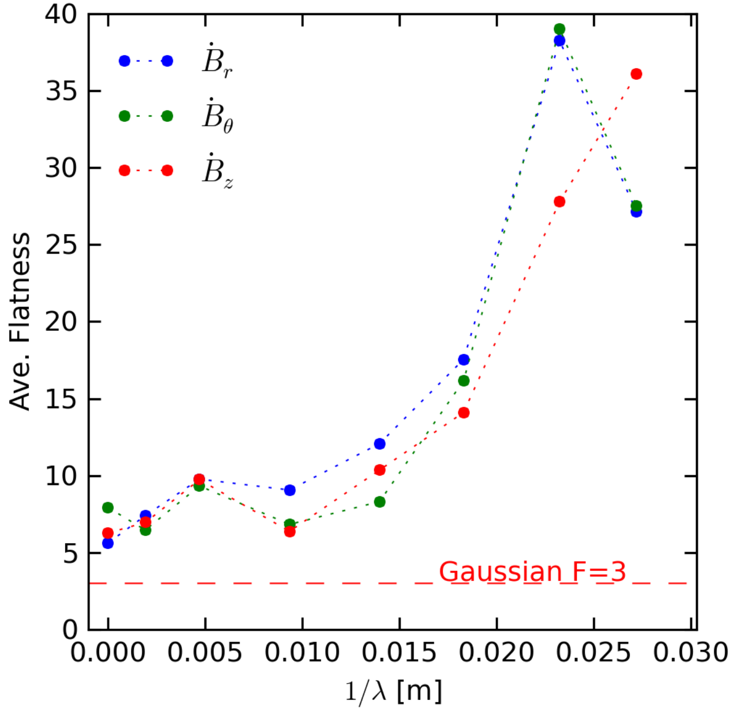


Flatness (intermittency) scales with injected helicity

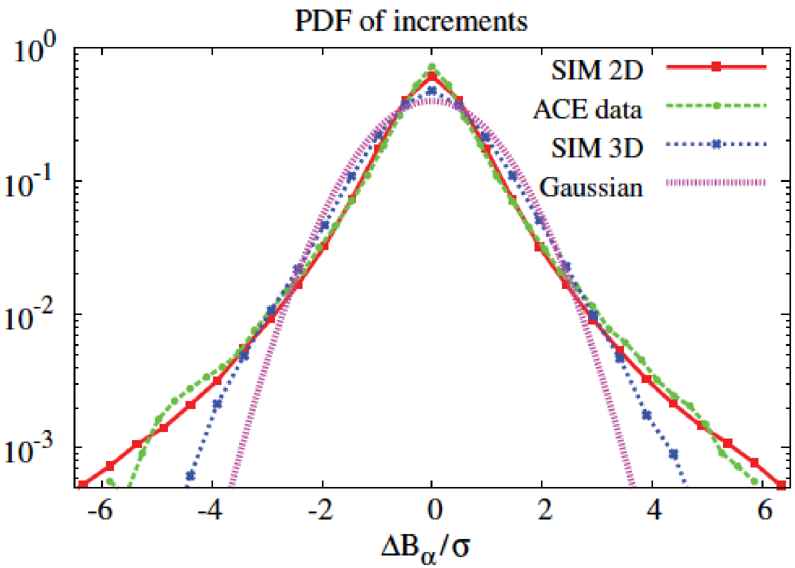


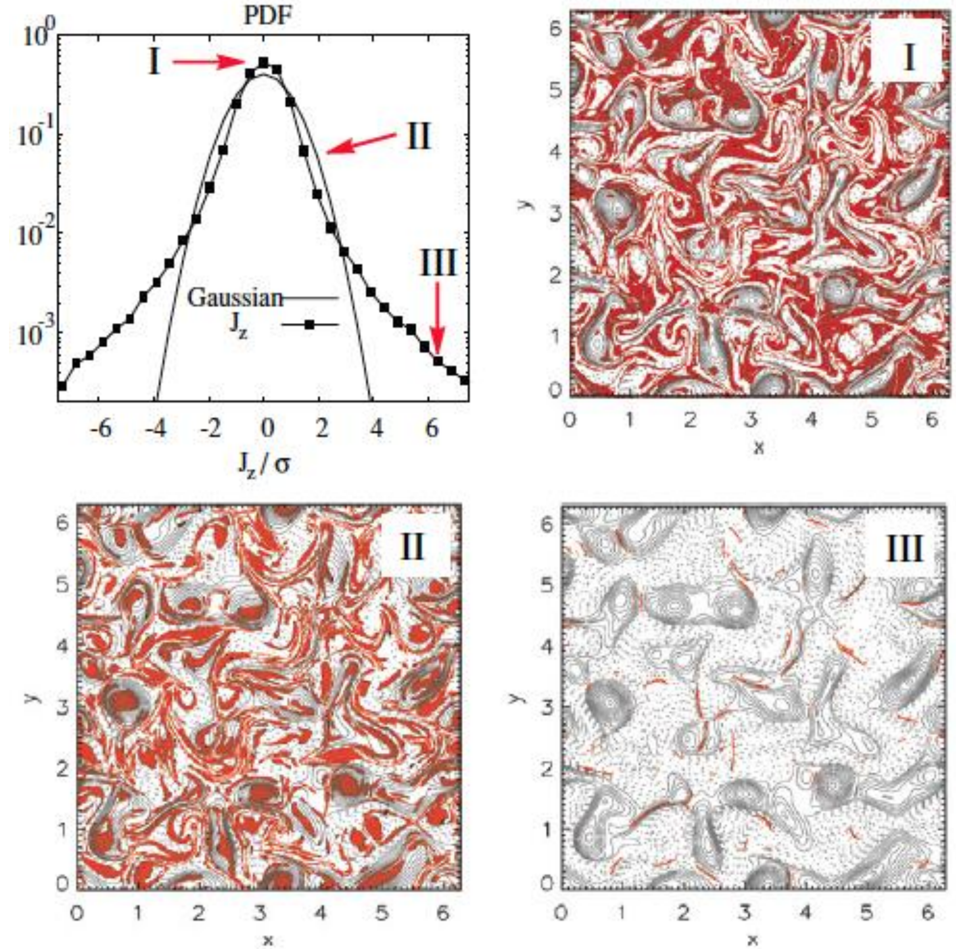


1) Resulting structure of turbulence is affected by amount of helicity.

2) A change in turbulent characteristic is seen in a higher order moment (4th order flatness), but not in 2nd order spectra.

Fat tails indicative of current sheets, reconnection?

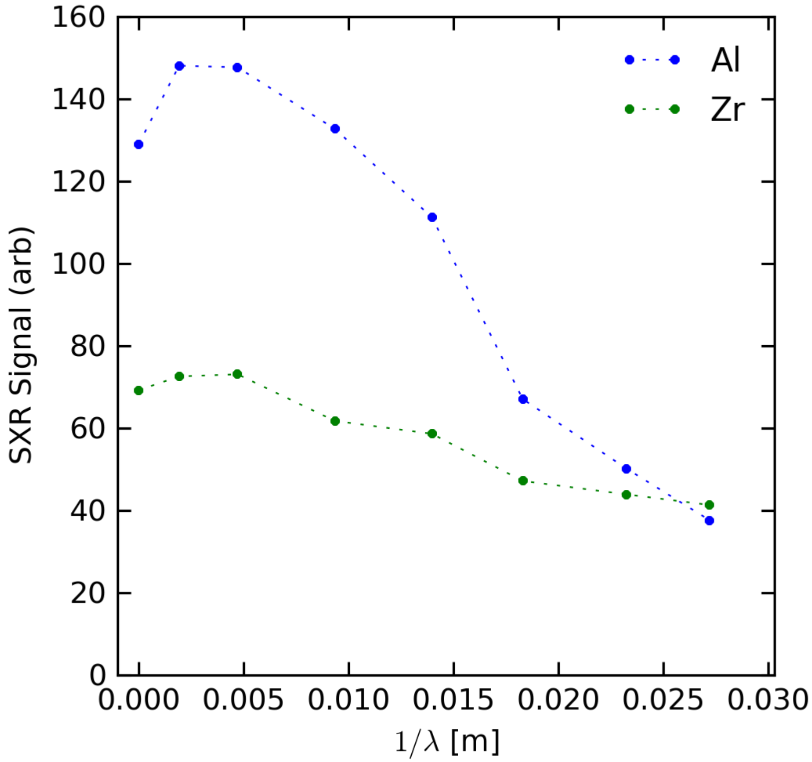


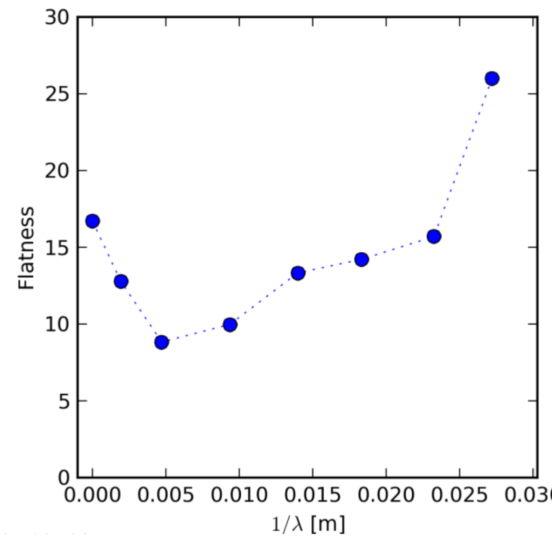


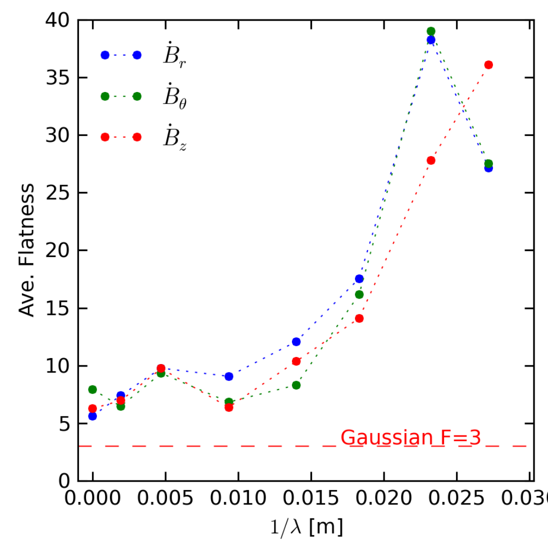
MHD simulations by Greco et al* suggest the intermittency observed in field signal is due to the presence of current sheets generated by reconnection sites

*Greco ApJ 2009

Ion temperature bursts scale with helicity, but integrated SXR signal scales inversely







Could structure size limit electron acceleration (thus, soft x-ray production)?
Higher structured turbulence resulting from more reconnection sites?

Conclusions

1) SSX plasma exhibits characteristics of a turbulent state—power-law spectra, intermittency
2) Measured B-field spectra indices steeper than Kolmogorov -5/3
3) Spectra indices unchanged by amount of injected helicity
4) Intermittency increased by higher injected helicity
5) Evidence for role of reconnection in turbulent structure observed, but simulation needed

Next Steps

1) Further study turbulence through autocorrelation, cross correlation, find the Taylor microscale
2) Observe possible dispersion relation in plasma—connection to energy transfer rate?
3) Make more comparison to simulation
4) Seek further evidence for connection between reconnection sites and intermittency—perhaps simulation necessary?

References

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